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Contract No. AF 33(615)-2215
Project No. 7064

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UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

FOREWORD

This final report was prepared by Dr. Antonio Ferri, Director of the Aerospace Laboratory and Astor Professor of Aerospace Sciences; and Dr. Victor Zakkay, Assistant Director of the Aerospace Laboratory and Professor of Aeronautics and Astronautics. It presents research carried out under Contract No. AF33(615)2215, "Boundary Layer Characteristics for Hypersonic Flow in the Presence of Mass Addition", Project No. 7064. This contract is administered by the Aerospace Research Laboratories, Hypersonic Laboratory, Office of Aerospace Research, United States Air Force.

ABSTRACT

Problems connected with viscous hypersonic flow with and without mass addition have been theoretically and experimentally investigated. Specific viscous flow phenomena investigated are: the boundary layer with large density gradients, the mixing in the presence of a wall, and the mixing of two fluids in the presence of pressure gradients. In the field of viscous flow with mass addition, nonsimilar, nonequilibrium, laminar boundary layers with surface ablation of subliming plastic materials have been investigated analytically. In addition, research has been performed in a spectrum of related fields.

The results of the above research have been reported in nineteen technical reports which have been published in the open literature.

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NOMENCLATURE

C_D	Drag Coefficient
C_L	Lift Coefficient
D	Characteristic Length
d	Characteristic Length
f_w	Injection Rate Transformed Stream Function
g(y)	Initial Profile Function
K	Crocco Function (Also denoted $\varkappa$, Ref. a)
L	Characteristic Length
M	Mach Number
$\dot{m}$	Mass Flow Rate
N_R	Reynolds Number
R_e	Reynolds Number
r	Radius
$r_{\frac{1}{2}}$	Wake Half Radius
S	Streamwise Lees Coordinate
T	Temperature
u*	Wall Friction Velocity (Ref. j)
u	Streamwise Velocity Component
v	Normal Velocity Component
w	Binormal Velocity Component
x	Streamwise Coordinate
y	Normal Coordinate
z	Binormal Coordinate
α	Angle of Attack
δ'	Thickness Defined by Equations 10 and 12

ϵ	Eddy Viscosity; Compressible Kinematic Eddy Viscosity Coefficient
$\bar{\epsilon}$	Incompressible Kinematic Eddy Viscosity Coefficient
η	Normal Lees Coordinate
κ	Constant in Diffusivity Model (Ref. p)
μ	Viscosity Coefficient
ν	Kinematic Viscosity Coefficient (= μ/ρ)
ξ	Transformed x Coordinate (Ref. d and f)
ρ	Density
τ	Shear
ψ	Stream Function (Ref. f)

Subscripts

a or A	Adiabatic Condition
B	Denotes Blasius Solution
c	Represents Cooling Length
e	External Stream Condition
i	Denotes Initial Station or Condition
j or J	Counting Subscript (= 1,2,3,...)
o	Initial Value or Stagnation Condition
s	Exact Similar Solution
S.L.	Stream Line
t	Denotes turbulent
w	Wall Condition
x	Derivative with respect to x
y	Derivative with respect to y
z	Derivative with respect to z

Supercripts

$(\ )'$	Denotes Derivative or Changed Variable
$(\dot{\ })$	Denotes Time Derivative

I. INTRODUCTION

Many of the basic fluid dynamic problems, which have received a great deal of attention in the lower speed range, are still unsolved or partially solved in the hypersonic speed range. Presently, such problems have become of great importance because of the long range interest on behalf of the Air Force to develop hypersonic vehicles. Flight at hypersonic speeds introduces many problems connected with aerodynamic heating, thereby imposing severe requirements for cooling the structure of the vehicle. In order to solve some of the heating problems, extensive use is made of ablating materials as well as localized cooling by means of injection. The purpose of this investigation is to obtain basic information in practical problems related to viscous flow phenomena in the hypersonic range. The investigations carried out are of both an experimental and a theoretical nature.

The statement of work for Contract AF33(615)2215 reads as follows:

1. Boundary layers along axially symmetric bodies will be investigated experimentally for several Reynolds numbers. Effects of localized injection of several fluids, both liquids and gases, will be considered.

2. The effect of ablating materials on boundary layer growth will be investigated with and without pressure gradients.

3. Mixing problems in the presence of large pressure gradients, produced by alteration of the geometry at the outer boundary, will be investigated.

4. The effect of entropy generators on heat transfer reduction at large Reynolds numbers will be investigated. Heat transfer, boundary layer profiles and pressure gradients will be measured in the tests.

5. An attempt will be made to develop a numerical analysis for the hypersonic laminar and turbulent boundary layers where the factors considered in the experimental work will be included. The results obtained from the experimental work will be used as a guide for such a development. A mixing analysis will be performed where radial as well as axial pressure gradients will be considered.

The principal investigators were Dr. Antonio Ferri, Director, Aerospace Laboratory and Astor Professor of Aerospace Sciences; Dr. Lu Ting, Professor of Aeronautics and Astronautics; and Dr. Victor Zakkay, Assistant Director, Aerospace Laboratory.

The actual work performed under the contract covers all items listed above with the exception of item 4. By enlarging the scope of work covered under item 1, item 4 has been replaced.

II. DESCRIPTION OF THE WORK PERFORMED IN THE DIFFERENT FIELDS

1. Boundary layers along axially symmetric and two dimensional bodies

In this phase of the work a number of interesting investigations dealing with approximate analytical and numerical solutions of the laminar and turbulent boundary layers have been completed.

a. Laminar boundary layers

Most of the boundary layer problems which are of practical interest cannot be treated by the method of similar soltuions. Further the integral method which has been used so extensively to yield solutions to nonsimilar problems is not amenable to the treatment of boundary layers with irregular initial profiles.

Approximate analytical solutions for nonsimilar boundary layers are presented in Ref. a. In this report the Crocco equation for zero pressure gradient and a constant viscosity density product is linearized and the results applied to boundary layer problems where the given initial profile is much different from the Blasius profile. Four linearization assumptions are used, each of which leads to an eigenvalue problem having a discrete spectrum of eigenvalues. This allows the shear stress to be expressed as a series. Far downstream of the initial station the first term of this series dominates. Solutions valid in this region are obtained for each assumption and are compared to the exact similar solution. The best assumptions are applied to two examples where it is desired to find the skin friction behind a region having mass injection. Comparison to finite difference solutions shows the present solution to be satisfactory and superior to solutions obtained by perturbations from the Blasius soltuion. The accuracy of the solution is improved by making multiple linearization assumptions in the streamwise direction and application is made to the problem of free mixing in the vicinity of a flat wall. Fig. 1 presents the results of a calculation performed with seven terms in the equation. The downstream variation of the parameter $(\bar{K}/\bar{K}_s)_w$, the non-dimensionalized Crocco parameter, is indicated. Also included are the results of Libby, Fox and Pallone. The circular points are obtained from the second order perturbation of Libby and Fox.

Comparison shows that the present results are in good agreement with the exact solution of Pallone. Also it may be noted that this agreement is better than the first order perturbation solution.

Therefore, this method has the advantages that (1) it can be used where the initial profile differs significantly from the Blasius profile, where the perturbation method cannot; and that (2) good agreement is obtained without second order corrections for problems where the perturbation method can be used.

A further discussion of perturbations for the same problem and asymptotic solutions in boundary layer theory is presented in Ref. b. For boundary layer solutions with zero pressure gradient the Blasius solution with any value of x_o for the initial station represents the leading term of asymptotic solutions. With this interpretation the Blasius solution will be the optimum one term solution if the value x_o is so chosen that the next term i.e., the first term of the perturbation solution vanishes; x_o is therefore defined by the condition that the deviation of the initial profile from the Blasius solution is orthogonal to the first eigenfunction of the perturbation equation.

In order to demonstrate the application of this method, Fig. 2 shows an initial profile which simulates the problem of free mixing in the vicinity of a flat wall. The velocity profile at the initial station $x = L$ is composed of a free mixing profile with velocity ratio .5014 on the top of a Blasius profile. Both the mixing layer and the boundary layer have been taken to start at $x = 0$. The initial profile $g(y)$ is monotonically increasing in y but the derivative $g'(y)$ first decreases to a positive minimum, then increases to a maximum and finally decreases to zero as y increases. The shearing stress at the wall is non-dimensionalized by the Blasius solutions with several values for x_o: $x_c = L$ or x_o

is defined by the matching of displacement thickness and momentum thickness; it is found that thicknesses are quite different from the numerical solution even for $x/L \sim 10$. It can be seen that the Blasius solution approaches the numerical solution rapidly for $x/L > 2$.

This clearly demonstrates the utility of defining x_o in a similar manner for a boundary layer problem; the Blasius solution above approaches the numerical solution by finite difference method soon after the initial station for various initial profiles.

Ref. c presents a method for maintaining a prescribed shear distribution over a surface. In several applications over some portion of a flight vehicle, it is desirable to maintain a constant shear and therefore a constant heat transfer below a given value. The solutions of such problems play an important role in improving the overall performance characteristics of such vehicles. One method of achieving this condition is by controlling the pressure gradient over the body. In a general way the problem is that of determining the pressure gradient required to obtain a specified skin friction, e.g., we may wish to know the magnitude of the gradient required to reduce the skin friction to a suitable value within a specified distance, and thereafter to maintain the skin friction constant at the reduced value. In the latter case, it may be of interest to determine the distance required for the boundary layer profiles to approximate similar profiles. Specifically then the problem posed is: given an initial boundary layer flow what is the pressure gradient required to obtain a specified wall shear downstream of the given initial station.

One of the earlier attempts to determine properties downstream from given initial conditions was that of Goldstein (Ref. 1) who assumed an initial velocity profile together with a free stream pressure gradient and calculated the wall shear for the compressible flow. Unfortunately, the convergence properties of his solution were not too good. This may have been caused by two factors in his analysis: a) a discontinuity was permitted in the wall shear at the initial station since the initial shear and pressure gradient were specified completely arbitrarily. In fact, the investigation of the behavior of the boundary layer in the neighborhood of such discontinuities was one of the purposes of the analysis and b) the initial velocity profile was specified as a power series in y, a variable with a range $0 < y < \infty$. It was therefore impossible to represent the entire profile by a small number of terms valid over the entire range of y. While the approach of Goldstein is maintained, the above difficulties are avoided in the present paper. The first is bypassed by allowing discontinuities in the shear derivatives, but not in the shear itself. This of course still covers a wide variety of applications, among them the determination of downstream profiles after fluid injection is abruptly terminated. The second difficulty is circumvented by working with the Crocco equation which employs the velocity as an independent variable and the shear as a dependent variable. Since the range of the dimensionless velocity is finite $0 \le u \le 1$ it becomes feasible to represent a wide variety of initial shear profiles by polynomials in u valid with a good degree of accuracy over the entire profile.

The solutions obtained were applied to the case of an initial profile typical of those encountered when fluid is injected vertically

into the boundary layer up to some station. Mach number distributions were then found which could maintain the wall shear constant at its initial value and a second distribution was found which would give a specified linear reduction in downstream wall shear. Results were obtained for initial free stream Mach numbers of $M_\infty = 1/2$ and $M_\infty = 2$. For the constant shear, the wall temperature and free stream Mach number variation in the subsonic case was found to be sufficiently close to the similarity solution for "local similarity" to hold and the shear profiles downstream of the initial station tended to approach the similarity shear profile for constant wall shear. At $M_\infty = 2$ the free stream Mach number and wall temperature variation differed markedly from the similarity distribution so that local similarity could be assumed and the downstream profiles could be compared with similarity solutions. In Figs. 3 and 4, we present the development of the profiles for both cases. In Fig. 5 the Mach number distribution required to maintain constant shear is indicated.

Due to the importance of the shear stress on the wall, it is sometimes convenient to simplify the problem by investigating the solution for the shearing stresses only and overlooking the determination of the complete velocity profile.

Ref. d starts with a simple approximation to the von Mises equation, leading directly to the heat conduction equation, the shear stress may be obtained in closed form as a function of a transformed coordinate ξ. The relation between ξ and the physical

coordinate x is then obtained for the Blasius problem in the following way. The Blasius profile at $x = x_o$ is used as an initial profile and the approximate solution for the shearing stress $\tau_B(\xi, x_o)$ is obtained as a function of ξ and the parameter x_o. By equating the approximate shearing stress to the Blasius solution, a relationship between ξ, x and x_o is obtained in the form $\xi/x_o = G(x/x_o)$. Making the further assumption that the transformation from ξ to x is a function of the local shearing stress only, the solution for a general initial profile and zero pressure gradient is obtained by matching the approximate solution $\tau(\xi)$ with the shearing stress $\tau_B(\xi, x_o)$ which corresponds to an initial Blasius profile at $x = x_o$. By using the preceding relationship between ξ/x_o and x/x_o, we can establish the correspondence between ξ and x. The application of the method to the more general case of flows with pressure gradient requires further investigation and here it has been limited to the problem of similar flow. The method has been tested for several types of profiles and the comparison with exact numerical solutions show good agreement not only for initial profiles close to Blasius profile but also for irregular profiles. A comparison of the present method with that of an exact solution is presented in Fig. 6. In this figure, the downstream variation of a non-dimensionalized shear distribution is shown and compared to the results obtained from the similar solutions by Cohen and Reshotko.

Some further considerations related to the perturbation type solutions presented in Refs 2 and 3 are discussed in Ref. e. Although not quantitatively indicated, these previous solutions

have some limitations with regard to the initial value problems. that can be treated. The initial profiles for, say, the momentum equation solutions must be close to Blasius; clearly this places restriction on the problems which can be handled with some degree of accuracy. Indeed even for the case of small deviations from the Blasius function, solutions presented in Ref. 2 indicate that resort had to be made to second order solutions to obtain reasonable results.

The techniques which can improve the accuracy of such initial value problems are investigated here. These methods make use of the fact that the origin of the Lees coordinates (s,η) is unknown. In previous papers it had been assumed that this origin and that of the initial profile were coincident. However, this assumption is not necessary and in some cases it appears undesirable. Further, due to the parabolic nature of the equations, the flow downstream can be completely specified by the initial profile without reference to its upstream history. The object of the report is the formulation of some suitable criteria permitting the determination of this origin and a discussion of the advantages of doing so.

Several applications are presented in the report. In these, the unknown origin was determined on the basis of equating displacement thicknesses of the initial and Blasius profile. This insures that the initial and final (Blasius) profile are close numerically. The problem treated was the downstream effect of upstream injection. Figs. 7 and 8 display the skin friction decays for two upstream injection rates, $-f_w = 0.5(2)^{-\frac{1}{2}}$ and $-f_w = (2)^{-\frac{1}{2}}$. Agreement with the exact solution of Pallone (Ref. 4) is seen to be excellent and

better than the previous approximate results. We may note that for the higher injection rate the original perturbation scheme could not even be applied. Also shown on the figures are results based on local similarity.

b. <u>Numerical technique in laminar boundary layers</u>

The choice of the von Mises (Ref.5) variables is a most natural one for a theory in which the smallness of the normal velocity component is almost indespensible. The advantages offered by these variables in connection with free mixing have been exploited in both analytical and numerical method of solutions. On the other hand, the existence of a wall singularity in the application of the transformations to flow fields in the presence of solid surfaces have caused many an investigator to abandon his efforts. [illegible] (Ref. 6) and Schlichting (Ref. 7) referred to it as the "unpleasant wall singularity". Approximate schemes utilizing these coordinates have been developed by von Karman and Millikan (Ref. 8) and much more recently, their method has been extended by Kosson (Ref. 9). A numerical method with special considerations to adverse pressure gradients in incompressible flows has been presented by Mitchell and Thomson (Ref. 10) and with some improvement later on by Mitchell (Ref. 11).

In the analysis considered in Ref.f , a numerical method based on the von Mises variables is applied to the laminar compressible homogeneous boundary layer with arbitrary pressure gradients. The method is based on a marching ahead technique utilizing an explicit numerical scheme, with a stability

criterion which is an adaptation of the one presented by Richtmyer (Ref.12) for linear parabolic equations.

In the neighborhood of the wall where the derivatives become unbounded, the numerical solution fails. For this region a series solution is introduced which, in conjunction with a set of compatibility conditions at the wall on the one hand, and the numerical solution adjacent to the wall region on the other, permits an accurate determination of the behavior of the dependent variables within the region as well as the value of the shear and heat transfer at the wall.

The comparison with other exact solutions clearly indicates that an explicit numerical method utilizing the von Mises variables provides an accurate and efficient method of solution to the compressible boundary layer equations.

Special care in using this system must be taken in regions close to the separation point. The difficulties associated with the solution of the boundary layer equations in the neighborhood of the separation point become magnified in the present variables. Due to the rapid divergence of the streamlines from the wall an accurate determination of the derivative at the wall becomes difficult once the streamline curvature becomes too large. In order to insure the validity of the results it is recommended that when a separation point is anticipated in the flow field, the calculations should be repeated at a smaller step size, and the convergence of the solution then verified. An alternative whereby a variable step size can be utilized near the wall which seems feasible from stability considerations has not been incorporated in the program.

The numerical scheme has been applied in detail to laminar slot injection problem. The solution of the single slot in a uniform flow indicated that the most significant parameters describing the wall temperature distribution are the stagnation temperature of the coolant and the mass flow of the coolant. The initial distributions of velocity and static temperature have their effects mostly as far as the skin friction coefficient and displacement thickness are concerned. Consequently they must be considered when transition is imminent. It was shown that independent of the initial profiles a cooling length designating the region on the wall where the adiabatic wall temperature remains below the stagnation temperature of the coolant, is well represented by the equation

$$x_c = 0.25 \frac{\rho_e u_e}{\mu_e} \left(\frac{\dot{m}_j}{\rho_e u_e}\right)^2$$

Following this region the temperature of the wall rapidly increases.

These results are shown in Fig. 9 and are plotted as T_{A_W}/T_{O_J} versus ξ/Ψ_J^2 (a non-dimensionalized stream wise coordinate).

The program using the finite scheme outlined in Ref. f together with the governing equations is presented in Ref. g. A flow chart, listings and detailed description of the necessary data input are also given.

c. Three dimensional laminar boundary layer

Some attempts to handle the three-dimensional boundary layer for certain classes of problems is prescribed in Ref. h. For a three dimensional boundary layer with x-coordinates along the inviscid stream lines on the surface of the body, it is well known that the four unknowns, the density ρ and the three components

of the velocity u, v, w are governed by the continuity equation, the x-component of the momentum equation, the energy equation, and a combination of y- and z- components of the momentum equation with the higher order pressure variation term eliminated. It is clear that the problem is not defined if only the initial profiles of density, ρ, and the velocity component parallel to the inviscid streamline, u, are specified. If four initial profiles, ρ, u, v and w are prescribed, which is likely the case in many real problems, these initial data in general will not be compatible with the governing differential equations. For the case when the two components of velocity v and w are much smaller than the third component, u is parallel to the inviscid streamline. It is pointed out that a compatibility condition on the four initial data: velocity components u_i, v_i, w_i and the density, ρ_i, exists while the enthalpy h_i is related to ρ_i by the equation of state.

The method is then applied to the analysis of the merging of a circular jet with a two dimensional boundary layer as shown in Fig. 10. It will be assumed that for $x < 0$ the interference between the jet and the boundary layer is negligible. The velocity components u, v, w and ρ at $x = 0^-$ are given by the linear superposition of the solution of a circular jet and that of a two dimensional boundary layer or a refined solution which includes the first order interference effect. To study the merging of the jet with the boundary layer for $x > 0$, it will be necessary to solve the three dimensional boundary layer equations by numerical integration. The proper initial data at $x = 0^+$ are given by the three invariant conditions,

$$u(x = 0^+) = u(x = 0^-), \quad \rho(x = 0^+) = \rho(x = 0^-),$$

$$\left\{ (\rho uv)_z - (\rho uw)_y \right\}_{x = 0^+}$$

$$= \left\{ (\rho uv)_z - (\rho uw)_y \right\}_{x = 0^-}.$$

In case that the solution at $x = 0^-$ is given by the circular jet solution and a two dimensional boundary layer solution without interference, it can be shown easily that $(\rho ux)_z - (\rho uw)_y$ is equal to zero for either solution. Hence, the third initial condition becomes

$$\left\{ (\rho uv)_z - (\rho uw)_y \right\}_{x = 0^+} = 0$$

The problem of the decay of a disturbance in an incompressible, zero pressure gradient flow is investigated in Ref. i. It is well accepted that in an incompressible, zero pressure gradient flow over a flat plate, any disturbance associated with the initial profile, far enough downstream, diffuses and the asymptotic solution, i.e. the Blasius similarity solution, becomes applicable with respect to a virtual origin. It is desired, however, to obtain a criterion that established, for a given disturbance, a characteristic length beyond which the difference between the exact solution and the locally similar solution becomes insignificantly small. "Insignificantly small" will be made more precise later in terms of percentage deviation. In a numerical analysis such an investigation can be carried out by considering a number of examples and then generating a correlation. Since necessarily such correlations are based on a finite number of numerical examples, any conclusions that follow must be qualified as "reasonable" rather than definitive.

In order to obtain the characteristic length for the attainment of local similarity, several initial profiles were taken, and the numerical solution of Ref. 13 were used in the correlation of the numerical data; a variety of disturbances were introduced in the flow.

Based on these results it is concluded that independently of the initial profile, the velocity field associated with a disturbance generated near the wall in a uniform flow becomes locally similar at

$$x_o = 1.0 \; \frac{\rho_e u_e}{\mu_e} \left(\frac{\dot{m}_j}{\rho_e u_e} \right)^2$$

d. Turbulent boundary layers

In the following section both theoretical and experimental investigations of turbulent boundary layers with and without mass addition are presented.

Ref. j has prescribed a generalized law of the wall and eddy viscosity model for wall boundary layers for regions of the flow field which depend only on a single space variable. The derivation is based on the assumption that the eddy viscosity law should vary continuously and smoothly from the logarithmic region of a constant shear flow to the immediate neighborhood of the wall where Reichardt's cubic power law is assumed to hold.

Considering the velocity as the independent variable and expressing ϵ as a function thereof yields, in the constant-shear case, a closed-form solution for the velocity distribution. The resulting distribution is shown to be in good agreement with experimental data. Included in the comparison are some recent

experiments where special care has been taken in order to obtain accurate measurements in the region very close to the wall.

By introducing a transformation, an extension of the preceding formulation to variable shear has been obtained. The generalized form has been then applied to the investigation of two problems: 1) the effect of Reynolds number (u^*d/ν) on the velocity distribution is a pipe and 2) the derivation of the law of the wall for boundary layers with injection or suction.

In the first problem, it has been demonstrated that, although to a very small extent, there is nonetheless a steepening effect on the slope of the logarithmic portion of the law of the wall with an increase in the Reynolds number. This result is in agreement with Hinze's analysis of Nikurudse's data. No explicit formulas for the Reynolds number dependence of the outer layers of the wall region have been obtained because the present solution constitutes, at most an "inner" solution. However, an expression for the thickness of a laminar sublayer defined in the text has been obtained which shows that ($y_{S.L.}/d$) is inversely proportional to the $\frac{3}{2}$ power of the Reynolds number (u^*d/ν).

In the second problem a law of the wall for injection or suction has been derived in terms of the parameter v_w^+. Good agreement with available experimental data has been indicated and a tendency for a decrease in the extent of the logarithmic portion of the law of the wall with increase of injection has been observed.

In Fig. 11 the law of the wall for injection for $v_w^+ = 0.45$ is compared to the data of Stevenson. It is clearly seen that good agreement exists between the present formulation and the experimental results.

The design of a vehicle capable of flying at hypersonic speeds requires a thorough understanding of the flow field characteristics about each component. One component that presents a major design problem is the inlet, which must produce a high compression ratio as well as good pressure recovery. For this purpose Ref. k presents an investigation of laminar, transitional and turbulent flows with adverse pressure gradient on a cone-flare at Mach 10, (Fig. 12). In this investigation, measurements of pressure, heat transfer and boundary layer thickness are made in regions of zero and adverse pressure gradient.

Several noses are fitted to the cone to produce blunted cone-flares, which are used to study separation. The experimental results are compared to some recent analytical investigations. The model used in this experiment consists of 7.5° half-angle cone blending smoothly into an axisymmetric flare body with a constant radius of curvature. The noses used to create the blunted cone-flares had radii of 0.26, 0.53 and 0.80 in. For laminar flow, the theoretical prediction gave good estimates of the heat transfer. A method employing "piecewise similarity" was found to give good results for estimating the boundary layer thickness. In the turbulent region, the simple flat-plate reference enthalpy method (FPRE) approximates the heat transfer rate well at lower

Reynolds numbers. Fig. 13 presents the heat transfer for both laminar and turbulent cases. Available theoretical methods were also found to give a fair estimate of the turbulent boundary layer thickness. For the condition of separated flow, an analytical analysis was possible in the case of incipient separation. The results showed that a small amount of blunting can lower the Reynolds number sufficiently to produce separation. The experimental results also show that a small amount of cross flow (produced by placing the model at small angles of attack) was capable of eliminating separation. Fig. 14 shows a condition of incipient separation.

It is further demonstrated that transition occurs at a value of $N_{R_\theta} \approx 430$ if the surface inviscid Mach number is close to the blunt nose surface Mach number. If transition occurs after the inviscid Mach number reaches the conical Mach number, then transition occurs at an $N_{R_\theta} \approx 700$.

In order to investigate the downstream effect of slot injection Ref. 4 employed the use of a center body designed as a streamline and inserted in a Mach 6 nozzle (see Fig. 15 a and b). In this manner, a thick turbulent boundary layer on the order of one inch was established ahead of the region where the gas was injected through the slot. Measurements of heat transfer distributions static pressure, and boundary layer profiles were made for various injection rates. A comparison was then made of the effect of upstream to downstream injection on the overall heat transfer characteristics to the body. Figs. 16 a and b present measurements for these two conditions. It is seen that the upstream injection, while having less of an effect, initially seems to persist over a much larger portion of the body downstream of the slot. Mach number boundary

layer profiles downstream have been measured and are presented in Figs. 17 a & b. These profiles are compared to those without injection. Finally, correlations for the adiabatic wall temperatures with injection rates are presented in Fig. 18. The skin friction coefficients have been obtained by the Reynolds analogy and by using the Libby-Baronti transformation in the incompressible plane. From the measurements, it is concluded that the boundary layer reaches its undistributed value after 1000 slot widths. The research on this phase of the work has not been completed, but it is expected to be accomplished by early 1968.

2. <u>Effect of ablating materials on bounday layer equations</u>

The investigation in this phase of the program resulted in two reports. The first (Ref. m) **formulates** the problem for the analysis of ablation at an axisymmetric stagnation point. In most of the literature, up to now, problems of ablation have been restricted to materials which sublime. These materials alter the boundary layer in a manner similar to that occuring in transpiration cooling and produce the well know "heat blockage" effect (Ref. 14).

It is of interest, in light of recent developments in materials technology, to study the additional features and possible advantages that may be associated with use of materials which first must melt and then vaporize before entering the boundary layer. A system of this type would then provide an additional layer, the liquid region, to enhance the reduction in heat transfer. Some previous studies have been performed for two layer models of the stagnation point boundary layer(Refs. 15-17). In these solutions, in order to obtain results which could be interpreted easily and which would not require large scale computations

simple chemical and transport models were assumed.

In view of recent calculations, demonstrating the relative importance of accurate transport properties (Ref. 18) and, recognizing that the associated boundary layer chemistry, for typical flow situations utilizing melting ablators, must be fairly complex, (T_e melting $= 2,000^oK$) it was desired to extend the previous analyses to account for these phenomena. Thus, there is studied in this investigation and formulated in general the stagnation point boundary layer solution for a melting and vaporizing material (see Fig. 19). The describing equations are written to account for an arbitrary number of species including the effects of multi-component diffusion where it is assumed that the gaseous phase is in chemical equilibrium. The ablating material can be taken to be either one which dissociates immediately upon vaporization or one which acts as an inert diluent.

Some comments are relevant here with regard to the location of the interface between the liquid and gaseous regions and the applicable matching conditions. It is necessary, from a simple balance of momentum and energy, to make continuous the following quantities: mass flux of each specie, temperature, shear, heat transfer and streamwise velocity. It is well known that ablating materials will introduce some mass into the boundary layer at all temperatures. Thus the question of the location of the interface depends not on the temperature level but on a balance between the net heat flux to the surface and the resultant mass transfer associated with this driving potential (Ref. 15). In Ref. 15, this problem was avoided by assuming that the vaporized material entering the boundary layer had the same properties as air. The gaseous and liquid regions were then solved independently for a spectrum of interface conditions and the matching was performed by inspection of the final results. To consider a more realistic problem here, the ablated

material is accounted for in the gaseous region and thus the coupled problem is considered.

The solution of the resulting differential equation may be carried out after transformation to the Levy-Lees (Ref. 19) variables (s,η). The transport properties of the multi-component mixture can be calculated by the methods of Ref. 20. The resulting ordinary differential equations for both layers form a two-point boundary value problem. To avoid the split nature of the boundary conditions, the equations are formally integrated and solved by an iterative procedure based on successive approximation. This technique was first presented by Weyl (Ref. 21) and later applied to fairly complex systems in Refs. 18 and 22 and found to be extremely successful.

A numerical example is also presented. There is discussed the ablation of silicon dioxide, S_iO_2 into an oxygen atmosphere. The properties of this material suggest its immediate dissociation (Ref. 23), upon vaporization, into S_iO and O_2. The gaseous region is taken to be in equilibrium, with O_2 and O as the reacting species and S_iO inert; thus three species are considered and the multicomponent nature of the flow exhibited.

This method of analysis could be applied with slight modification to the injection of a liquid in the stagnation point region.

In the second of these reports (Ref. n), an analysis of nonsimilar, nonequilibrium solutions of the laminar boundary layer equations with surface ablation of subliming plastic materials is performed. The hypersonic flow over a body that is thermally protected by an ablative material results in an important interaction between the boundary layer and the ablating surface. The mechanism by which they are coupled depends,

in general, on the type of ablative process, i.e., evaporative, melting, charring, etc., as well as the mechanical and thermodynamic properties of the material, e.g. thermal conductivity and ablation temperature. The primary mechanisms can be put in two broad categories, namely: chemical and thermal. For example, in the case of carbonaceous materials which are characterized by high ablation temperature ($T > 3000°K$) the coupling with the boundary layer occurs primarily by heterogeneous, surface oxidation reactions. In this case the local ablation rate can be completely independent of the local heat transfer rate. On the other hand, materials that ablate at low temperature ($T < 1000°K$) do not generally oxidize (heterogeneously) to a significant degree. However, the local ablation rate in this case is directly proportional to the convective heat transfer rate (and inversely to the heat of ablation of the material). This thermal coupling mechanism is formalized by consideration of a detailed balance between the various contributions to the surface heat transfer and mass transfer rates. Thermal coupling results in a locally self-regulating ablation rate, that is independent of the specific vehicle heat shield design (e.g., material depth) and transient effects (e.g., trajectory, heat history), after ablation has begun, if the thermal conductivity of the material is sufficiently low. This description is appropriate for several plastic materials, namely: polytetrafluorethylene (Teflon) and polyformaldehyde (Celcon or Delrin), that receive particular attention in subsequent parts of this report. These plastics can be classified as subliming ablators. Their chemical heat of ablation is relatively small (about 1000 cal/gm), resulting in large rates of injection of ablated gases into the boundary layer with attendent

reductions in heat transfer and skin friction, (Refs. 24, 14). Significant changes in the flow field also occur due to reaction of the ablated gases in the boundary layer, (Refs. 14,26). However, large departures from chemical equilibrium are to be expected at hypersonic conditions if the density is low enough to permit laminar flow,(Ref.26). (The Reynolds and Damkohler numbers are closely related.) Therefore, consideration is also given to nonequilibrium reactions involving the multicomponent mixture of primary constituents of air and the ablation products.

The occurrence of nonequilibrium reactions in the flow field generally introduces a nonsimilar dependence of the flow field properties on the spatial coordinates, although self-similar solutions do obtain under certain restricted conditions (Refs. 26,28,29). The boundary conditions associated with the abovementioned cases of a low temperature ablator of low thermal conductivity are only weakly influenced by nonequilibrium effects, and they may therefore be approximately similar. However, the change in boundary conditions due to a discontinuous change from an ablating condition at the surface to nonablating conditions, or vice-versa, results in an essentially nonsimilar behavior, regardless of the occurrence of reactions. The appropriate boundary conditions for the general, nonsimilar case is considered in this investigation.

Teflon and Celcon or Delrin are chosen as representative of this class of ablators. The coupling between the ablation process and the boundary layer is discussed, and the governing equations and boundary conditions are presented for a multicomponent, reacting mixture of ablated gases and the constituents of air. The multistrip integral method of solving the equations is developed in detail.

The transport properties of the chemical species, including the binary diffusion coefficients, in the C/F/O/N system associated with the pyrolysis and oxidation of tetrafluoroethylene (C_2F_4) in air, and in the C/H/O/N system due to oxidation of hydrogen and carbon monoxide [the assumed products of formaldehyde (CH_2O) pyrolysis] in air, will be presented in a subsequent report. The results of representative calculations for ablating Teflon cones and Celcon or Delrin cones at hypersonic conditions will also be given later.

3. Mixing with and without pressure gradients

The problem of turbulent mixing in wakes and between two streams will be discussed in this section. Some insight into the characteristics of the turbulent transport properties will be discussed initially, and then applied to hypersonic turbulent wakes. Finally, a review of heterogeneous mixing problems and observations on mixing inside channels will be discussed.

An investigation of the radial variation of the eddy viscosity in compressible turbulent jet flows is performed in Ref. 6. Although our knowledge of the actual transport mechanism of turbulent mixing processes is still rather limited, recent experimental and theoretical work has provided more insight into the problem and helped clarify several points which were open to question until now. For example, it could be shown that Prandtl's expression for the incompressible turbulent kinematic viscosity coefficient $\bar{\epsilon}$ is incorrect for a jet exhausting into a stream with uniform velocity (Ref. 30). A comparison of theoretical predictions with experimental data (Ref. 30) indicated errors of more than 100 percent when the aforementioned

relation for $\bar{\epsilon}$ was used. However, good agreement between theory and experiments was obtained when $\bar{\epsilon}$ was taken as $\bar{\epsilon} = \kappa(\rho u)_{\mathbb{C}} r_{\frac{1}{2}}$.

Ref. 31 suggested a modification of Prandtl's expression for compressible flows. Instead of relating the kinematic viscosity coefficient ϵ to the velocity difference, it was proposed to relate the dynamic coefficient $\mu_t = \rho\epsilon$ to the maximum mass flux difference. This model yields good results for jets exhausting into a quiescent atmosphere; however, it fails for jets exhausting into a stream with uniform velocity. Similarly for incompressible jets, theory and experiments agree quite favorably when the mass flux difference is replaced by the mass flux at the centerline (Ref. 30). The formulation of Ref. 31 is discussed in detail in Refs. 32 and 33. In Ref. 33, it was shown that for a jet exhausting into a quiescent atmosphere the same result can be obtained by linearizing the momentum equation.

An expression which relates the incompressible and the compressible kinematic viscosity coefficient directly has already been given by Ting and Libby in 1960 (Ref. 34). The expression they proposed includes the radial variation of ϵ caused by the density gradients in the mixing region. A comparison to other formulations or experimental data has not yet been made.

It is the purpose of the present analysis to compare the aforementioned transformation with relations developed in Ref. 30. In that reference the streamwise momentum equation is solved for ϵ such that if all flow quantities appearing the expression for ϵ are determined experimentally ϵ and μ_t can be calculated. Data so obtained may serve to confirm the validity of already existing formulations for turbulent transport coefficients.

In Ref. 34 Ting and Libby showed that the turbulent compressible viscosity coefficient ϵ can be related to its incompressible value. Employing the Mager transformation (Ref. 35) and assuming that the moment about the axis of turbulent shear stress over an infinitesimal mass is preserved, the authors derived the following relation for the case of axisymmetric jets

$$\frac{\epsilon}{\bar{\epsilon}} = \frac{2\rho_o}{(r\rho)^2} \int_0^r \rho r' \, dr', \tag{1}$$

where ρ_o denotes a reference density. If ρ_o is chosen to be equal to $\rho_{\mathbb{C}}$, then the centerline values of ϵ and $\bar{\epsilon}$ are formally the same since

$$\lim_{r \to 0} \frac{\epsilon_{\mathbb{C}}}{\bar{\epsilon}} = 1 \tag{2}$$

If now $\bar{\epsilon}$ is specified, the compressible value of ϵ can be calculated provided the density profiles in the mixing region are known.

A different approach is taken in Ref. 30; there the momentum equation is solved for ϵ.

$$\epsilon = \left[\int_0^r \frac{\partial}{\partial x} (\rho u^2) \, r' \, dr' + \rho \, uvr \right] \left[\rho r \frac{\partial u}{\partial r} \right]^{-1} \tag{3}$$

Eq. 3 is considerably simplified as r approaches zero. Then ϵ reduces to

$$\lim_{r \to 0} \epsilon_{\mathbb{C}} = \epsilon = \left[\frac{u \frac{\partial u}{\partial x}}{\frac{2\partial^2 u}{\partial r^2}} \right]_{\mathbb{C}} \tag{4}$$

Values of ϵ and $\epsilon/\epsilon_{\mathbb{C}}$ may now be evaluated from Eqs. 3 and 4 provided the density and velocity profiles and their derivatives with respect to the

streamwise and radial coordinate can be determined. If this information is available, results of Eqs. 3 and 4 may serve to prove the validity of Eq. 1. Now ϵ and $\epsilon/\epsilon_{\not{c}}$ may be found by numerical integration of Eqs. 1 and 3, where the transverse velocity component v in Eq. 3 is evaluated from integration of the continuity equation

$$v = -\frac{1}{\rho r}\int_0^r \frac{\partial}{\partial x}(\rho u)r'\,dr' \tag{5}$$

The results of such an analysis showed that the kinematic viscosity coefficient ϵ varies appreciably in the radial direction, whereas the dynamic coefficient $\rho\epsilon$ does not exhibit such a strong radial dependence. This result confirms that the assumption of $\rho\epsilon$ remaining constant in the radial direction and varying in the streamwise direction as discussed in Refs. 31-33 is a reasonable approximation to its actual behavior. It was also shown that the viscosity coefficients ϵ and $\rho\epsilon$ do not strongly depend on the initial velocity ratio of the two streams. Fig. 20 presents the radial variation of the nondimensionalized turbulent viscosity coefficient $\rho\epsilon/(\rho\epsilon)_{\not{c}}$ for a hydrogen air mixture. It is seen that $\rho\epsilon/(\rho\epsilon)_{\not{c}}$ remains fairly constant until $r/r_{\frac{1}{2}} \approx 1$, and then varies considerably towards the outer mixing region.

An experimental and theoretical investigation of the turbulent far wake is performed in Ref. 9. The trail left by a body flying at hypersonic speeds has been, and is, of considerable interest. This is in part because of its implications for discrimination techniques of re-entry bodies. When the flow in the far wake region is laminar, and since the diffusion processes are well defined, standard techniques are available for prediction of the flow field. However, when the flow is turbulent, the transport properties are not well defined, and some assumptions need to be made

to define, e.g., a turbulent viscosity for use in the laminar equations in order to solve the problem.

Correlations of available theoretical analyses with experimental results have been scarce because of the significant difficulty encountered in measuring detailed flow properties in the far wake. Furthermore, these treatments themselves have been limited, since little is known about the transport mechanism. Therefore, various models for the viscosity have been chosen by different investigations and each seems to predict a different wake growth; up to now most of the analyses have been compared with experimental results obtained from Schlieren pictures or shadowgraphs of bodies fired in ballistic ranges. Since these comparisons have been made only of gross effects, it would, therefore, seem desirable to obtain more detailed measurements.

In this investigation, an approach similar to that used in problems of mixing is used in analyzing the flow field in the far wake. First, in order to understand the turbulent transport properties, a theoretical and an experimental study was conducted at hypersonic speeds and at a relatively low temperature, so that no dissociation or chemical reaction is present to complicate the problem further. Tests were performed at Mach 12 and at a stagnation temperature just high enough to prevent condensation. Since the present problem is analogous to that of mixing, a similar approach is applied. Detailed measurements in the flow field of Mach number, velocity and stagnation temperature were made between x/D of 10 and 600 for various model diameters.

The experiments were conducted at a stagnation pressure of 450 psia and a stagnation temperature of 2000°R. The Reynolds number based on freestream conditions is 4.5×10^5/ft; the model wall-to-stagnation-temperature ratio was 0.265. A flat-faced cylindrical model was chosen for the investigation.

The important experimental results are indicated in Figs. 21-24. In the first three radial profiles of velocity, Mach number and temperature are presented at distances downstream corresponding to 50, 100, 150 diameters, respectively. It is to be noted that more data is available; these are presented since they are typical. In Fig. 24 the complete centerline Mach number distribution is indicated.

The measured profiles are used as input data in the solution of the set of boundary layer equations. These equations have been set up in an implicit finite-difference scheme for an IBM 7094 computer. With these measured profiles, a straightforward marching downstream of the solution is obtained after an assumption of the form of the eddy viscosity The final profiles obtained from the machine are compared to the ones obtained from the experiments. Thus, a systematic evaluation of various models can be made. In the results, it is shown that the model that best describes the wake is that which also describes coaxial mixing. The viscosity selected for analysis were:

Model 1

$$\mu = \kappa \rho_{\mathrm{CL}} u_{\mathrm{CL}} r_{\frac{1}{2}} \tag{6}$$

Model 2

$$\mu = \kappa (\rho_e u_e - \rho_{\mathrm{CL}} u_{\mathrm{CL}}) r_{\frac{1}{2}} \tag{7}$$

Model 2A

$$\mu = \kappa\rho_e(u_e - u_{\text{℄}})r_{\frac{1}{2}}$$

Model 3

$$\mu = \kappa\rho_{\text{℄}}u_{\text{℄}}r_{\frac{1}{2}}\left[\left(\frac{2}{\rho y^2}\right)\int_0^y \rho y dy\right]$$

Model 4

$$\mu = \kappa\rho_{\text{℄}}(u_e - u_{\text{℄}})\delta'$$

where $r_{\frac{1}{2}}$ is defined as that value of y where

$$\rho u = (\tfrac{1}{2})\ (\rho_e u_e + \rho_{\text{℄}} u_{\text{℄}})$$

and where δ' is defined as that value of η,

$$\eta^2 = 2\int \frac{\rho}{\rho_e}\, y dy$$

where

$$(u_e - u)/(u_e - u_{\text{℄}}) = 0.01$$

Typical of the comparison between the analytic results and the experimental data is given in Figs. 25 and 26 for the centerline Mach number decays and profile prediction. The effect of turbulent intermittancy on the results is indicated in Fig. 27. Here the intermittancy was taken to be simply a radial modification of the eddy viscosity. While these results are not definitive they may be regarded as indicative of the importance including such intermittancy factors.

The mixing region of two semi-infinite parallel streams (Fig. 28) is analyzed by the boundary layer equations in Ref. q. However due to the missing third boundary condition, the location of the interface is undetermined to the order of the thickness of the mixing layer (Ref. 36). The missing third boundary condition can be recovered from the compatibility condition for the first order solution (Ref. 37) i.e. the balance of the first order induced pressure across the mixing layer. It has been pointed out in Ref. 37 that this compatibility condition becomes an identity and fails to yield the third boundary condition for the special case when both streams are subsonic and the zeroth order solution in the mixing region is represented by a similar solution. For this special case, it becomes necessary to look into the second order solutions and to derive the third boundary condition by integration of the second order normal component of the equation of motion across the mixing layer. This was carried out in Ref. 37 only for the simple case of incompressible flow. In this investigation the integration of the momentum equation will be carried out in a different manner so that explicit results can be obtained without the assumption of incompressibility for the mixing region of two subsonic parallel streams. This completes the determination of the interface for the mixing of two parallel streams.

Some observations of heterogeneous mixing inside channels is given in Ref. r.

The motion of compressible fluids in ducts has been investigated in great detail for the case of homogeneous flows. The classical procedure for the analysis is to consider the main part of the flow as inviscid, and then to introduce corrections on the boundary conditions at the

walls due to the displacement thickness produced by the skin friction and heat transfer at the walls. Such effects usually are determined on the basis of the pressure distribution obtained from the inviscid calculations. Following such an approach an iterative process can be introduced when the viscous effects are important. One-dimensional considerations are ususally effective for the determination of first approximations of choking problems in such flows, and choking criteria for such flows are well established.

The problem is more complex when either nonuniform flows, or flows having chemical reactions are considered. Such types of flows are of interest, in general, in propulsion systems and in diffusers for wind tunnels. The purpose of this report is to describe some of the interaction effects between the mixing and pressure distribution of a heterogeneous flow in a channel. In the discussion the viscous effects at the wall are ignored because such effects can be treated in a fashion similar to the homogeneous flow.

Criteria for determining the variation of pressure as a function of the characteristics of the two streams as well as the choking conditions due to mixing are determined. Depending on the initial conditions, the variation of pressure along the tube can be either positive or negative (see Fig. 29).

These results may be obtained from simple one-dimensional consideration which have been derived in Ref. r.

In conclusion for this section, a critical review of the heterogeneous mixing problems is performed in Ref. s.

The problem of mixing of two streams is extremely complex. Classical

aerodynamics has investigated for many years the problem of mixing of two coaxial gas streams. However, the quantitative understanding of such physical phenomenon is still incomplete even for laminar mixing where the transport properties are known, and the progress made both by theoretical and experimental investigation is still not even satisfactory.

From an analytical point of view, the difficulty arises because of the simplification introduced in the equations. In addition for turbulent mixing, the lack of knowledge of transport properties increases the difficulties of the problem. Classically, mixing problems are analyzed by introducing all simplications used in boundary layer theory. Unfortunately, the introduction of such simplifications in mixing problems are not necessarily justified and acceptable for the determination of important physical properties of the phenomena. Several questions can be raised with respect to the use of the boundary layer equations in mixing problems. The introduction of the boundary layer equations in the analysis of flow near a wall permits elimination of the second momentum equation. It can be shown that this simplification is not important in the determination of the variation of momentum along streamlines, and therefore of the skin friction which is one of the important quantities to be determined in boundary layer analysis. However, in mixing problems, the most interesting information required from the analysis is the rate of diffusion of a fluid of one stream into the fluid of the other, and such motion is controlled by the second momentum equation. As a consequence, the use of the boundary layer equations can be justified only when the characterisitics of the two streams (velocity, density, composition-temperature) are not too different.

If the two streams have physical properties which are widely different, then the scale for the determination of the order of magnitude, which is the thickness of the layer affected by the flow, becomes extremely large. Typical indication of the inadequacy of the approximation of boundary layer theory is given by the results with this approximation for a jet discharging into a quiescent atmosphere. In this case, in the external flow the velocity component normal to the axis of the jet is larger than the tangential component in the proximity of the origin of mixing; therefore, the description of the motion of the external flow cannot be adequately represented by the simplified equations. The normal component of the velocity is directly related to the mass of the flow entrained by the jet, and therefore is a quantity that must be determined with satisfactory precision.

When the approximation of the boundary layer theory cannot be justified for the analysis of mixing problems, then, as a consequence, the boundary conditions for a mixing problem cannot be specified in a simple form at the station where mixing begins if the two streams are subsonic. In this case, the mixing process induces velocity components which are significant upstream of the initial mixing region.

For the same reason it is very difficult to perform significant mixing experiments when the two streams are quite different. In this case, the characteristics of the experimental apparatus upstream of the mixing region have important effects on the mixing phenomena near the origin of mixing. This point is often neglected in presenting experimental results and in comparing experimental results. When the two streams are subsonic, even for the case of the mixing of two low velocity streams of the same fluid,

the initial conditions cannot be defined by giving the average properties of the two streams, i.e. the average velocities measured in terms of stream tube area and mass flow. These are defined only if the complete velocity distribution and pressure distribution at the station at the beginning of the mixing are given. Such distributions can change significantly from experimenta to experiment because they are affected by the experimental apparatus upstream of the mixing region; therefore presentation of experimental data on the basis of simple parameters such as average velocity ratio or average velocity difference of the two jets is not justified because such terms do not define completely either physical properties or the experiment.

From these considerations it appears evident that the problem of mixing of heterogeneous gases is far from being understood quantitatively. The analytical approaches available are, in many cases, not satisfactory; the experimental data are insufficinet, and often not accurately presented. Many new phenomena, some characteristic of the heterogeneity of the two streams are present because of the production of adverse pressure gradients, which even if small, often can strongly change the flow. Therefore, it can be concluded that

1. In the analysis of mixing, boundary layer types of equations are justified only when small gradients are present.
2. In experimental investigations, the disturbances in wind tunnels are of primary importance on the velocity profile for axially symmetric flow. Small disturbances produced at the walls can produce large variation in velocity decay. Fig. 30 shows the variation of Mach number at the axis of an axially symmetric jet, when an infinitesimal disturbance is produced at the wall of the jet.

3. The best quantity to measure is concentration and not velocity or temperature.

4. Several other equations are derived for hypersonic mixing.

III. DISCUSSION OF FUTURE RESEARCH

The studies which have been discussed in the preceding sections resulted in a large numer of interesting problems covering a broad spectrum of research areas. Some of these investigations have been preliminary in nature, and it would be advantageous to continue, in part, some of this, particularly the areas of practical interest.

In addition some of the advanced facilities at N.Y.U. have become operational and therefore basic experiments at high Mach number high Reynolds number can now be performed. On this basis a program of investigation of hypersonic flow at high Reynolds number comparable to flight conditions is now feasible.

1. The problem of mixing in the presence of a wall.

This problem is of importance in the study of tangential injection in supersonic streams and in the study of practical cooling systems. This investigation has already been initiated under the previous Contract AF33(615)2215, and it is proposed to continue this investigation. Experiments have been conducted at Mach 6 and detailed measurements of profiles have been made with slot injection. The tests have been conducted with air, and it is planned to use different gases to see the effect of varying the molecular weight of the gases. Included here will be measurements of concentration.

2. Viscous flow problems related to mixing with or without pressure gradient and mixing of a three-dimensional nature.

Up to now mixing in two-dimensional streams as well as coaxial streams has been analyzed and a prediction of the flow field in the mixing region is feasible. However, for more practical purposes the mixing could be enhanced considerably by considering asymmetric jets. In this a crossflow component will be present which could alter the flow field considerably. Therefore it is proposed here to analyze and perform experiments for three-dimenionsal jet configurations and study the mixing, as well as the turbulent transport properties under these conditions.

3. Viscous flow effects on separation of slender bodies at angle of attack and determination of $dC_L/d\alpha$ and C_D at high Reynolds number.

During the past two years the N.Y.U. Aerospace Laboratory has developed a high Reynolds number high Mach number facility which is capable of attaining R_e/ft of the order of 4×10^7 at a free stream Mach number of 12. This facility is especailly useful for studying viscous flow effects on separation over slender bodies since a variation of Reynolds number over the entire laminar, transitional and turbulent region could be attained in this facility. It is proposed to study these separated regions over slender bodies at an angle of attack in detail. In addition the lift and drag characteristics of various angles of attack will be measured with a three component balance.

4. <u>The effect on drag and lift of axially symmetric bodies due to ablation and injection at high Reynolds number</u>.

Current trends in the design of thermal protection systems for hypersonic vehicles,i.e. ballistic reentry vehicles, decoys, cruise vehciles, etc., have place attention on several "soft" plastic materials, e.g,, polytetraflourethylene (Teflon) and polyformaldehyde (Celcon or Deflon). These materials ablate at relatively low temperatures, 700° to 1000°K, and inject gaseous products directly into the boundary layer at relatively high rates.

The problem will be attacked within the framework of boundary layer theory using nonsimilar methods developed in earlier studies. The appropriate boundary conditions for the multicomponent problem have already been developed under the previous Contract. Therefore, it is proposed to continue this research and make a detailed analysis of the laminar boundary layer on an ablating surface. This is of special interest for communications, detection and discrimination.

5. <u>Analysis of viscous and inviscid three-dimensional flow fields over bodies of practical interest</u>.

Up to now there are no theories which can sufficiently account for the prediction of the flow field over slender bodies (i.e., cones) at angles of attack. The problem is difficult since the analysis must take into account the viscous separation which occurs on the low pressure side of the body. Such an analysis could be performed provided some insight is given by some experimental results. Under Item (3) of this proposal, such an investigation has been suggested. Therefore, in this part it is proposed to utilize some of the experimental results as a guide in order to develop the three-dimenionsal analysis.

6. Effects of entropy layer on laminar turbulent transition.

The reentry phase of ballistic and earth orbital missions can be characterized, in regard to transition from laminar to turbulent flow in the boundary layer on the vehicle, by a free stream Mach number of 20 to 25, by large streamwise gradients of Mach number and Reynolds number due to nose bluntness, and by aximuthal asymmetry and cross flow due to angle of attack (particularly if the vehicle is a lifting body). Recent experiments (Ref. 39) have demonstrated that bluntness and angle of attack have pronounced effects on transition at Mach number 5.5, which were not evident at Mach number 3. Moreover, the behavior found in these experiments indicate several trends of importance to vehicle design considerations, namely: the effect of bluntness at hypersonic Mach numbers (> 5) is such that the possibility of finding an optimum nose bluntness for maximum transition distance from the vehicle leading edge clearly exists, and the asymmetric transition distance effects due to angle of attack (including the potential degradation of the vehicle dynamic stability in the transition regime (Ref. 39) is reduced appreciably by increased nose bluntness.

IV. CONCLUDING REMARKS

The work performed at the N.Y.U. Aerospace Laboratory under Contract AF33(615)2215 has been reviewed. A broad spectrum of investigations bearing on high speed flow problems of current interest, has been indicated. A large number of interesting results have been obtained as exhibited by the attached list of nineteen technical publications prepared under the

subject contract. In particular, understanding has been gained in several fundamental problem areas related to the hypersonic boundary layer with mass addition. Finally, a continued research effort in problems related to mixing with and without pressure gradients, turbulent transport properties, viscous flow effects on the separation of slender bodies at angle of attack and at high Reynolds number and three dimensional viscous and inviscid flow fields is proposed.

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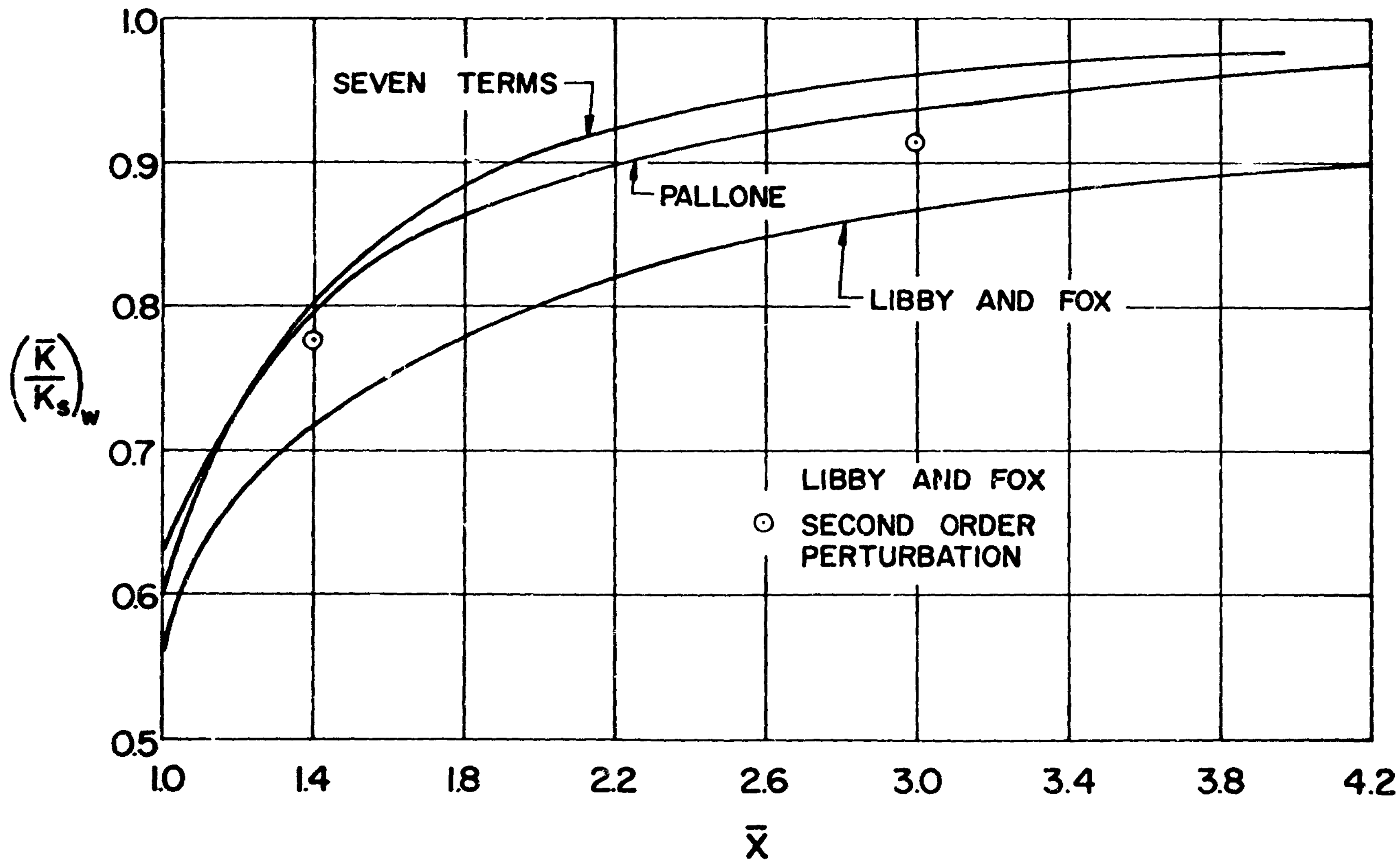


Fig. 1 Skin Friction Comparison ($f_w = -0.5$)

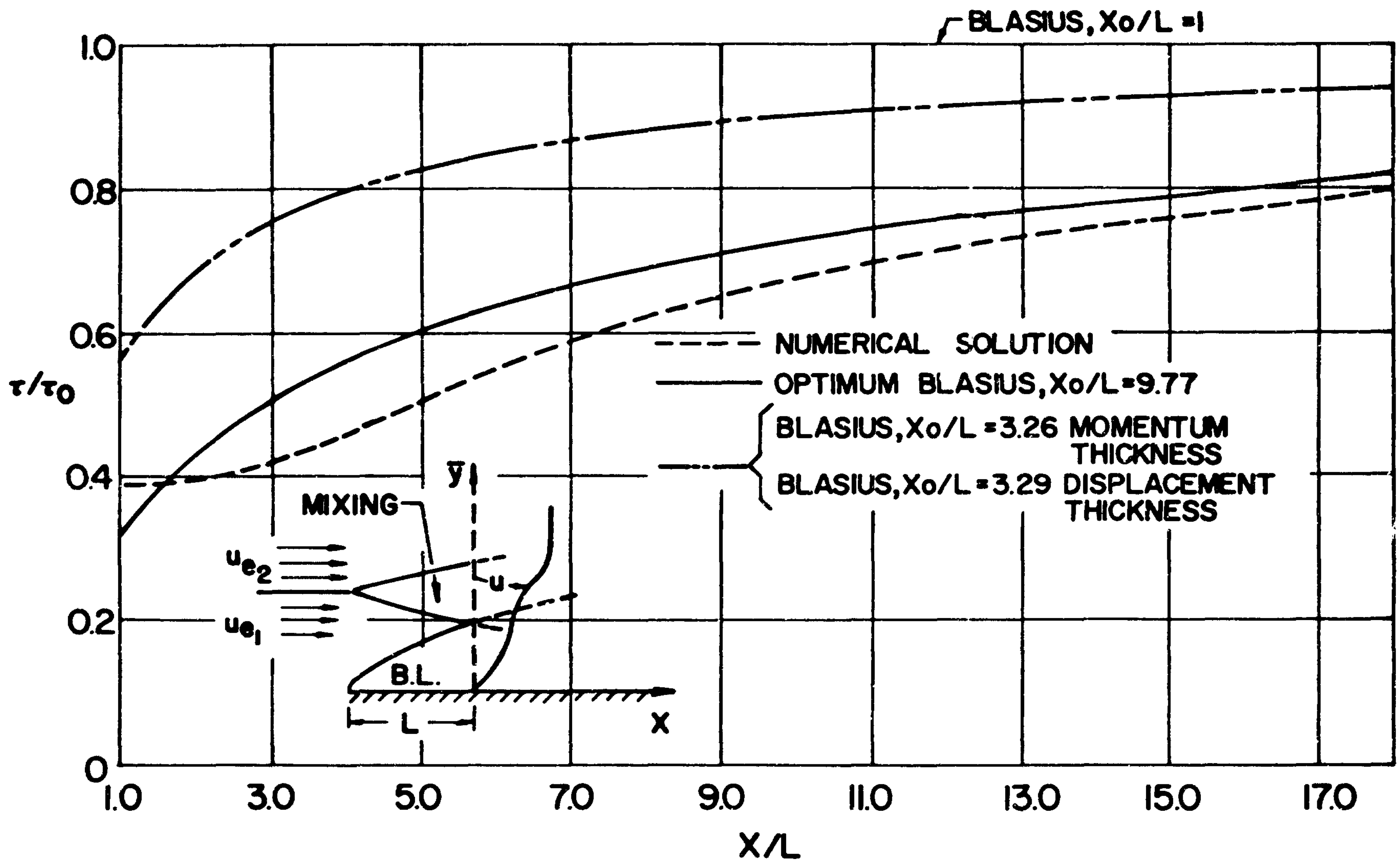


Fig. 2 Variation of Skin Friction for Mixing Boundary Layer Interaction

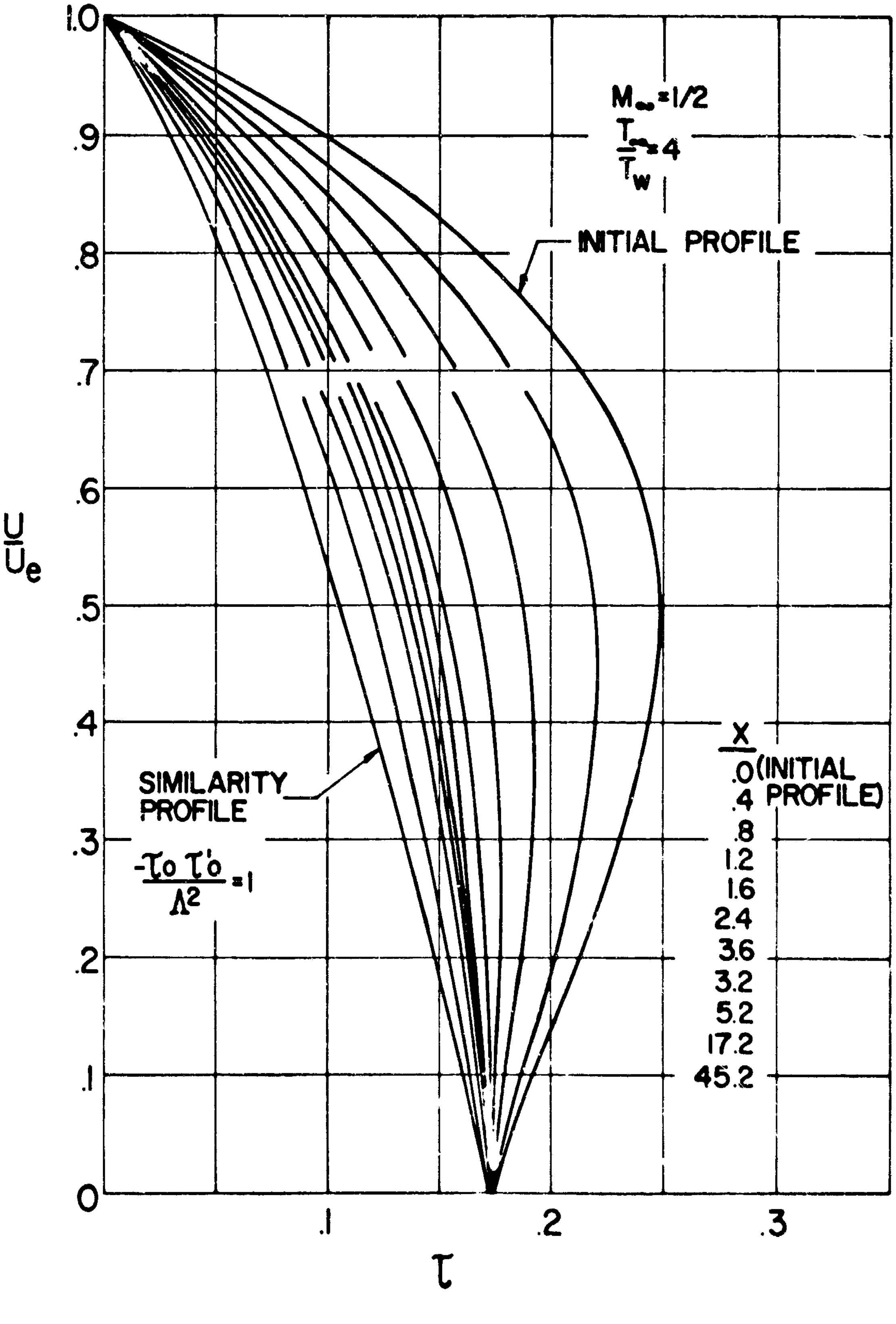


Fig. 3 Shear Profiles for Constant Wall Shear

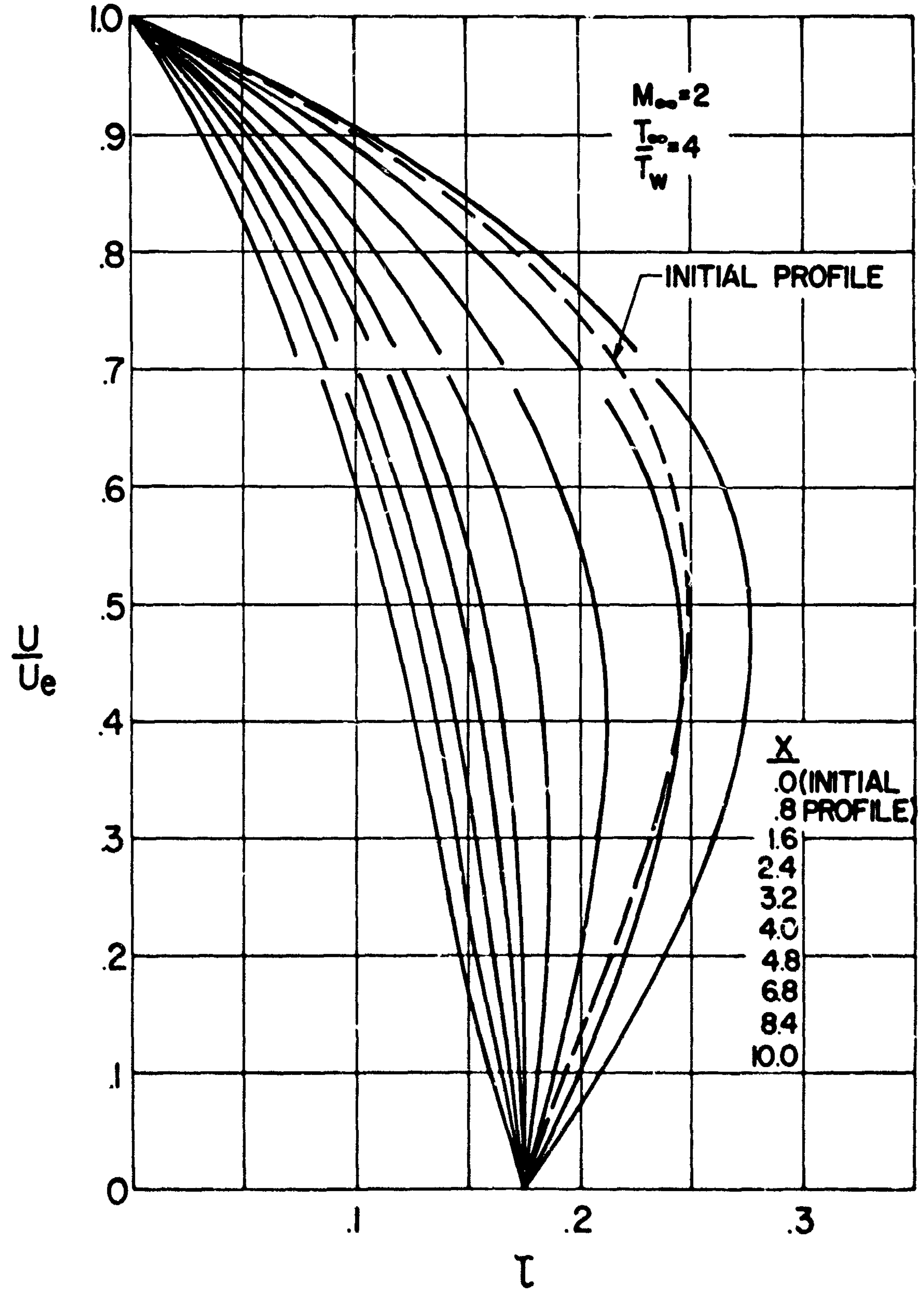


Fig. 4 Shear Profiles for Constant Wall Shear

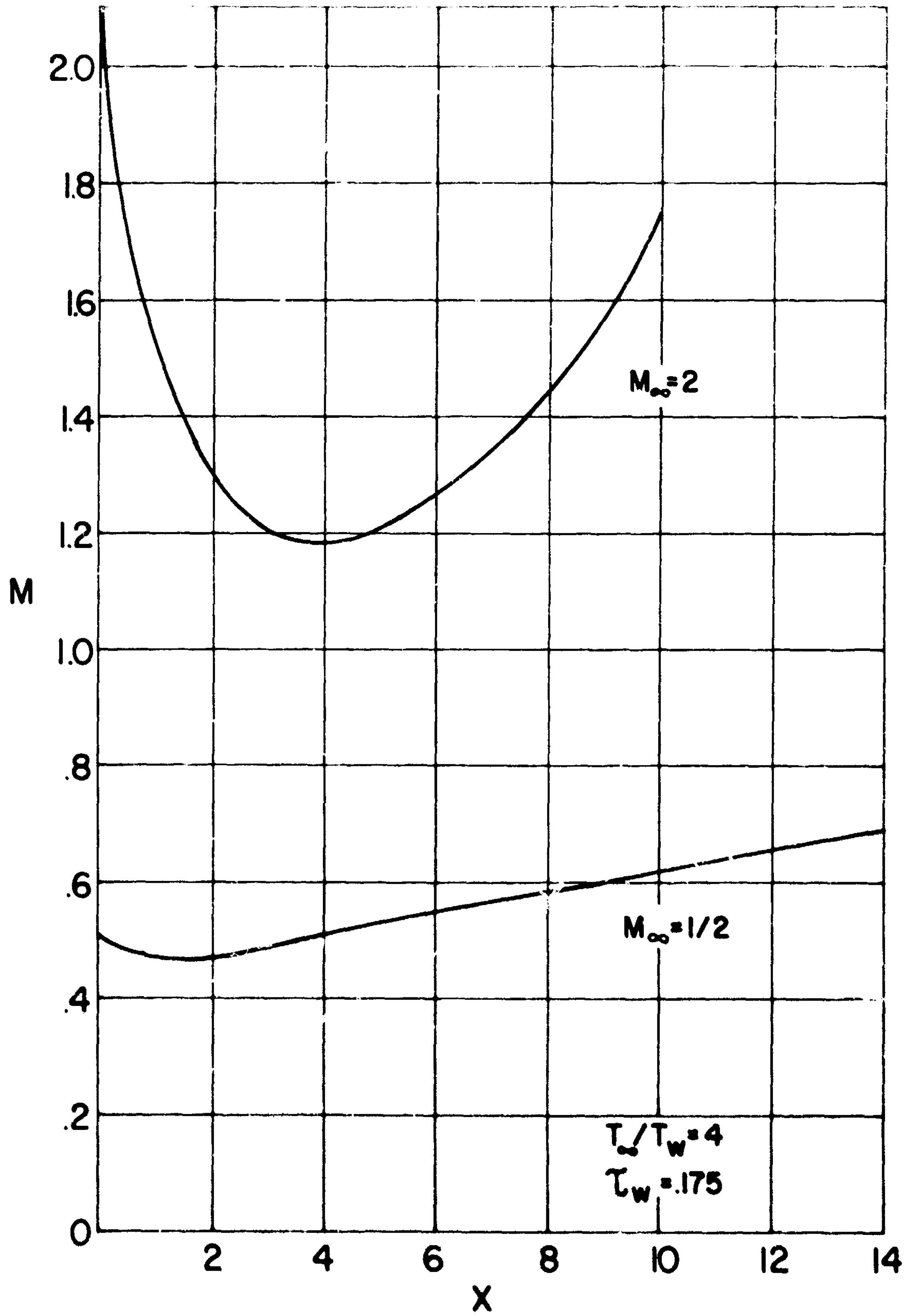


Fig. 5 Free Stream Mach Number Distribution for Constant Wall Shear

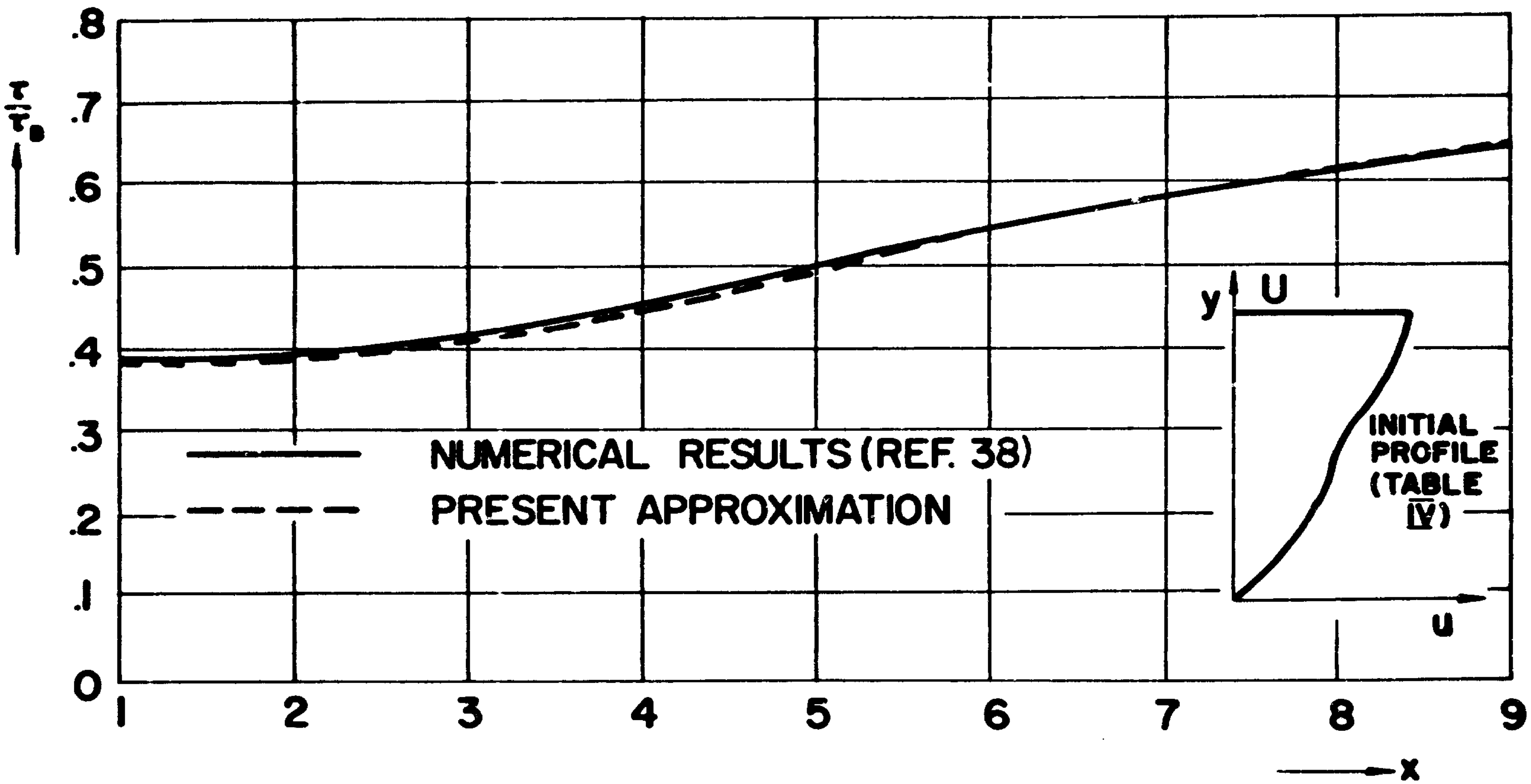


Fig. 6 Comparison of Shear Stress Distribution

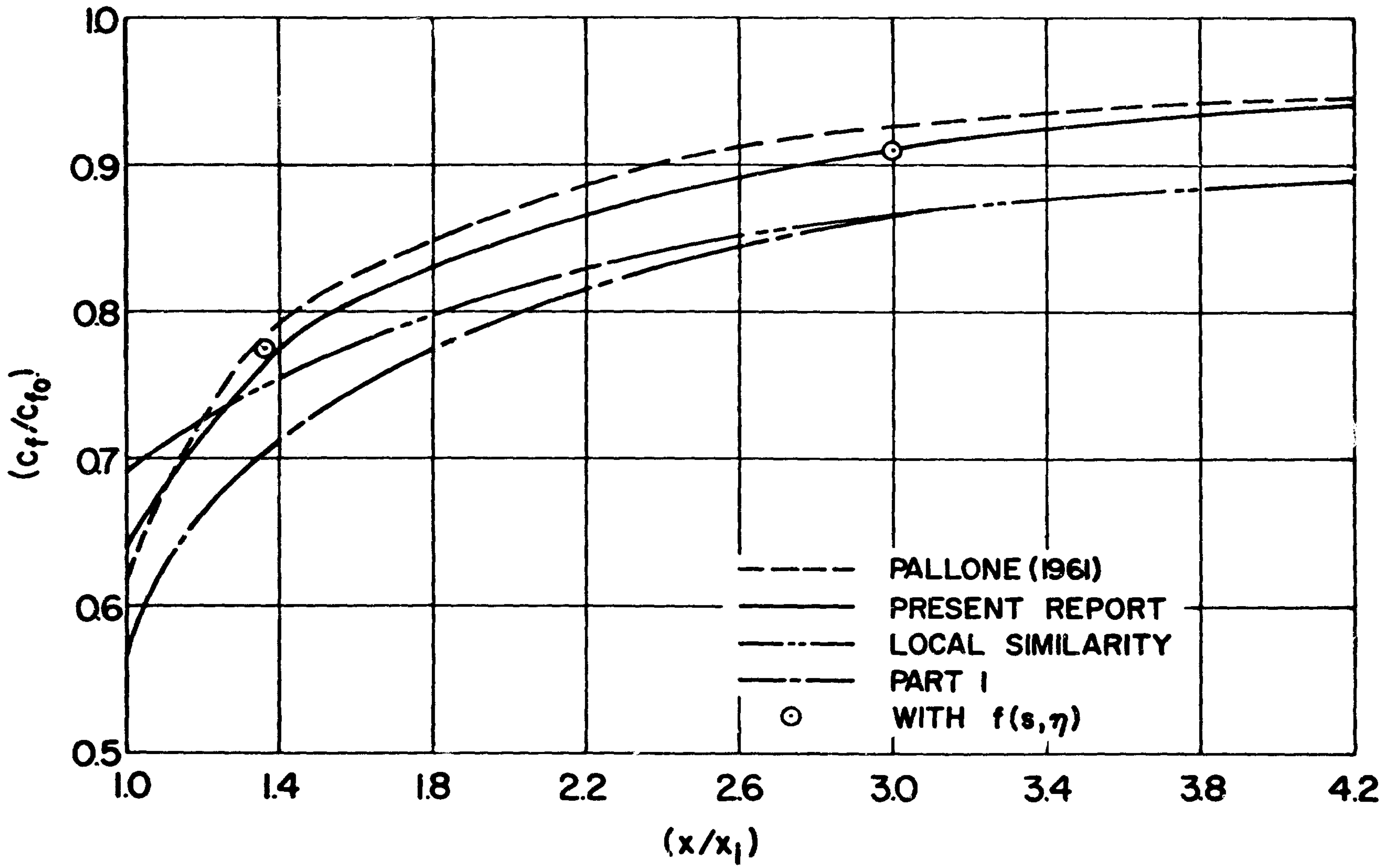


Fig. 7 Distribution of Skin Friction ($f_w = -0.5(2)^{-\frac{1}{2}}$)

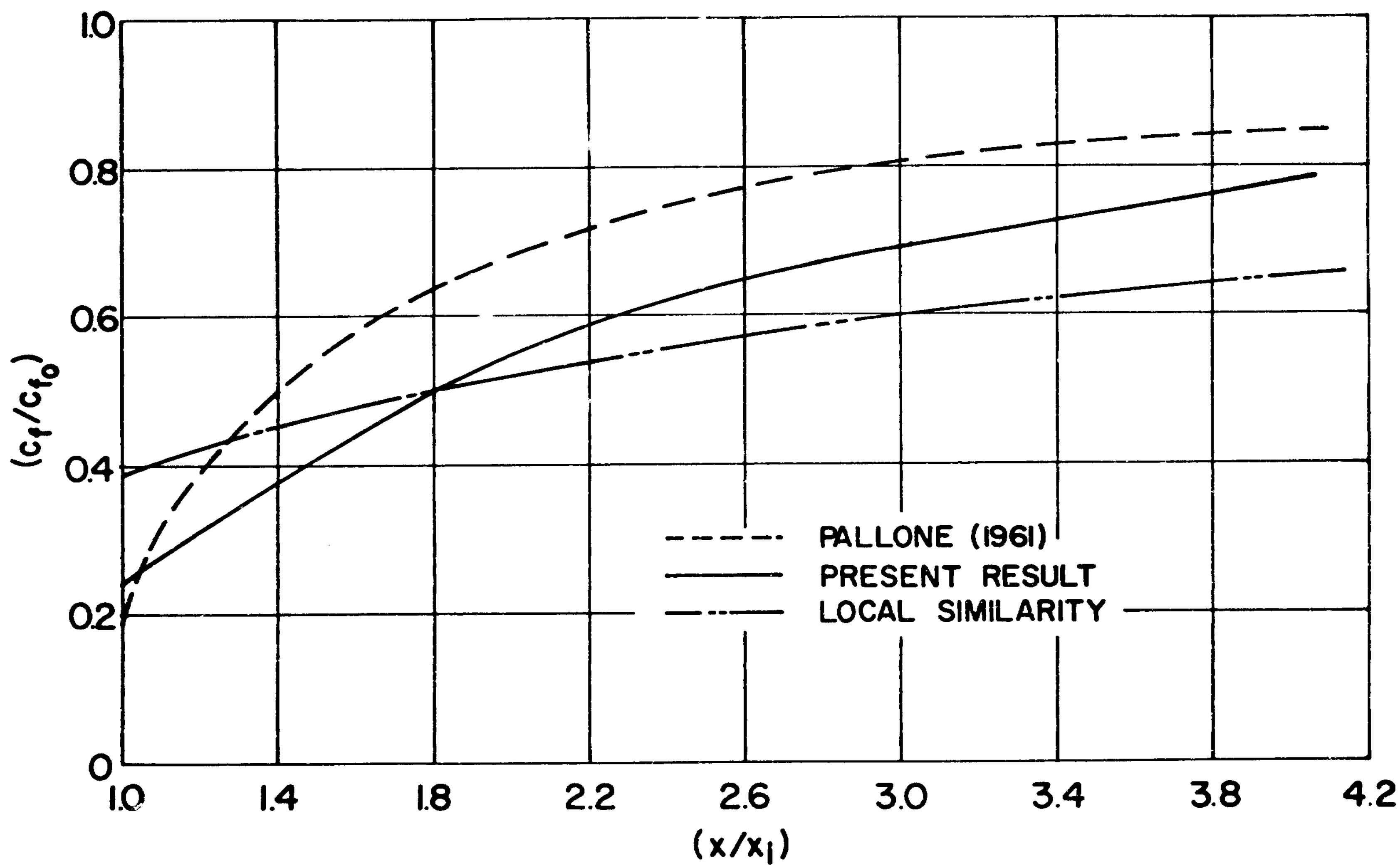


Fig. 8 Distribution of Skin Friction ($f_w = -(2)^{-\frac{1}{2}}$

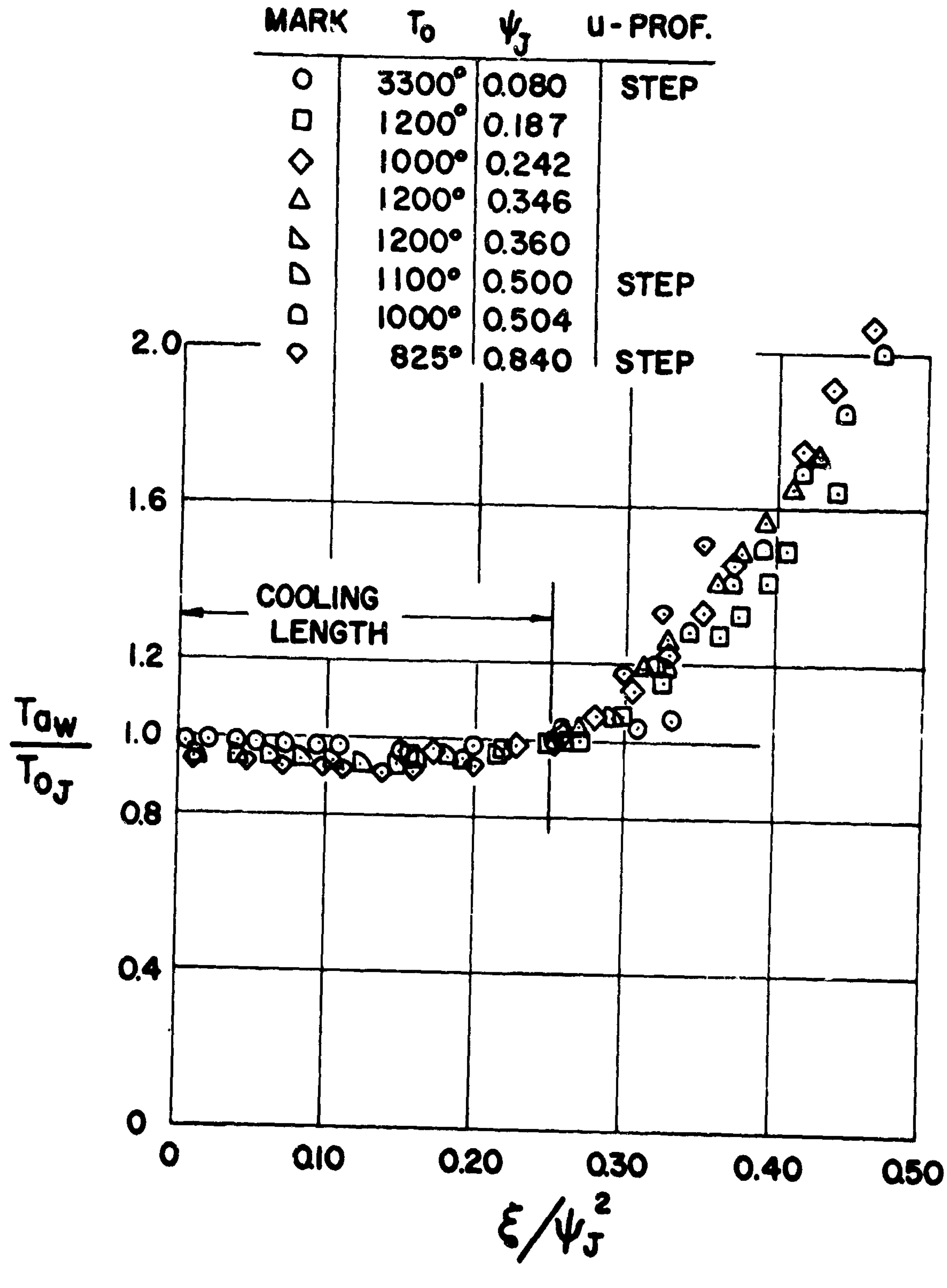


Fig. 9 Adiabatic Wall Temperature for Various Injection Rates and Different Coolant Stagnation Temperatures

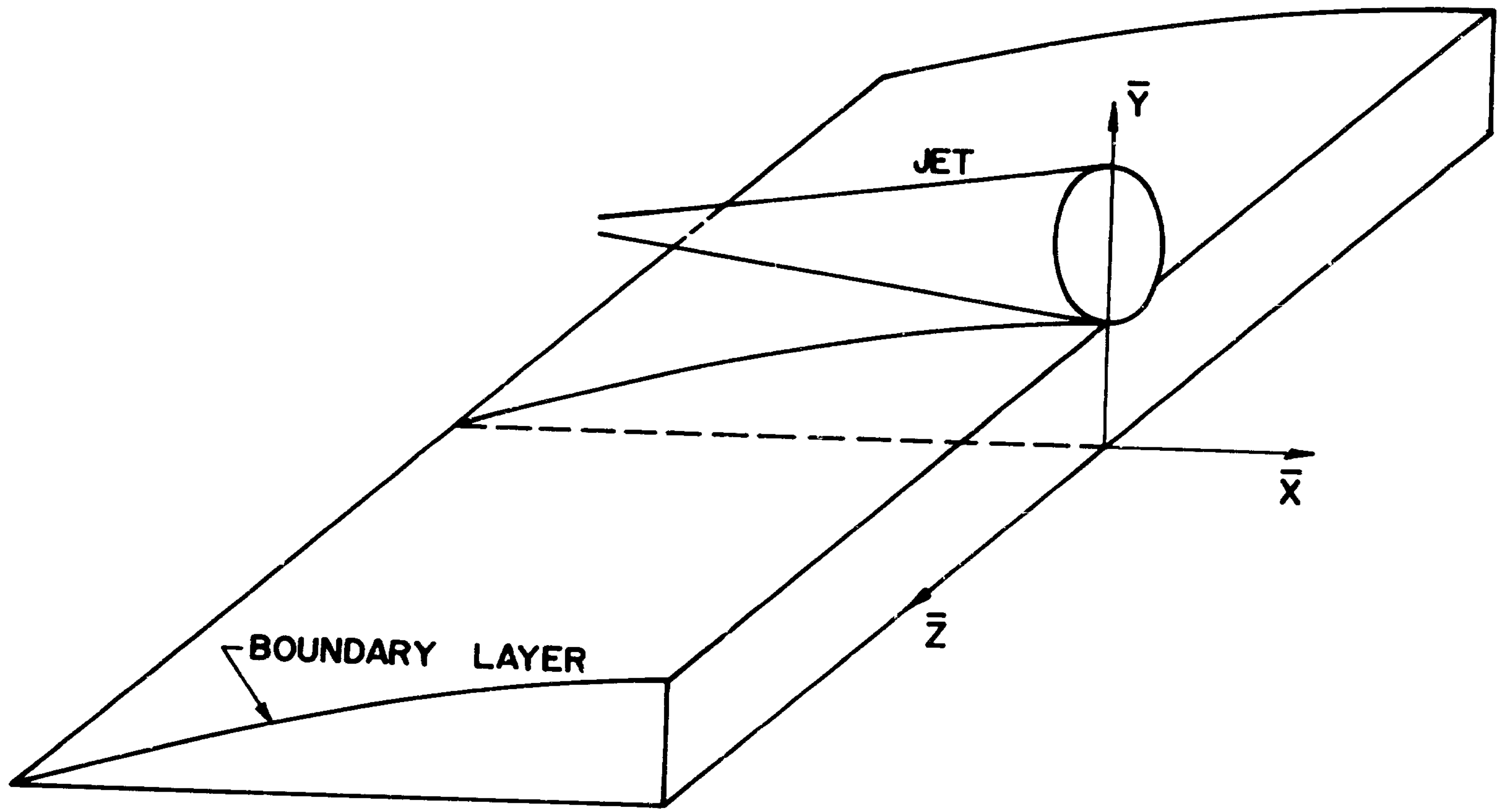


Fig. 10 Merging of a Circular Jet with a Two Dimensional Boundary Layer

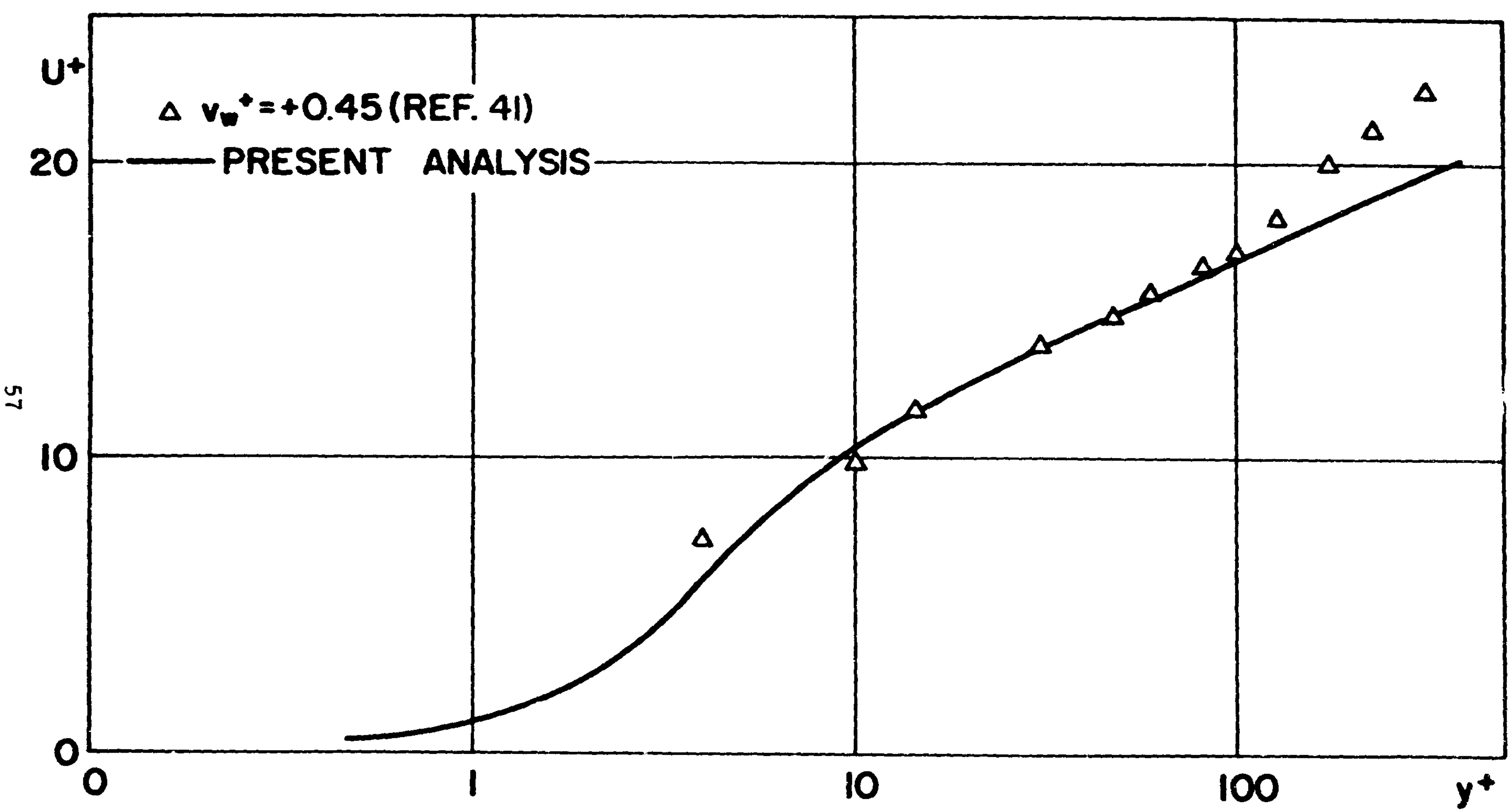
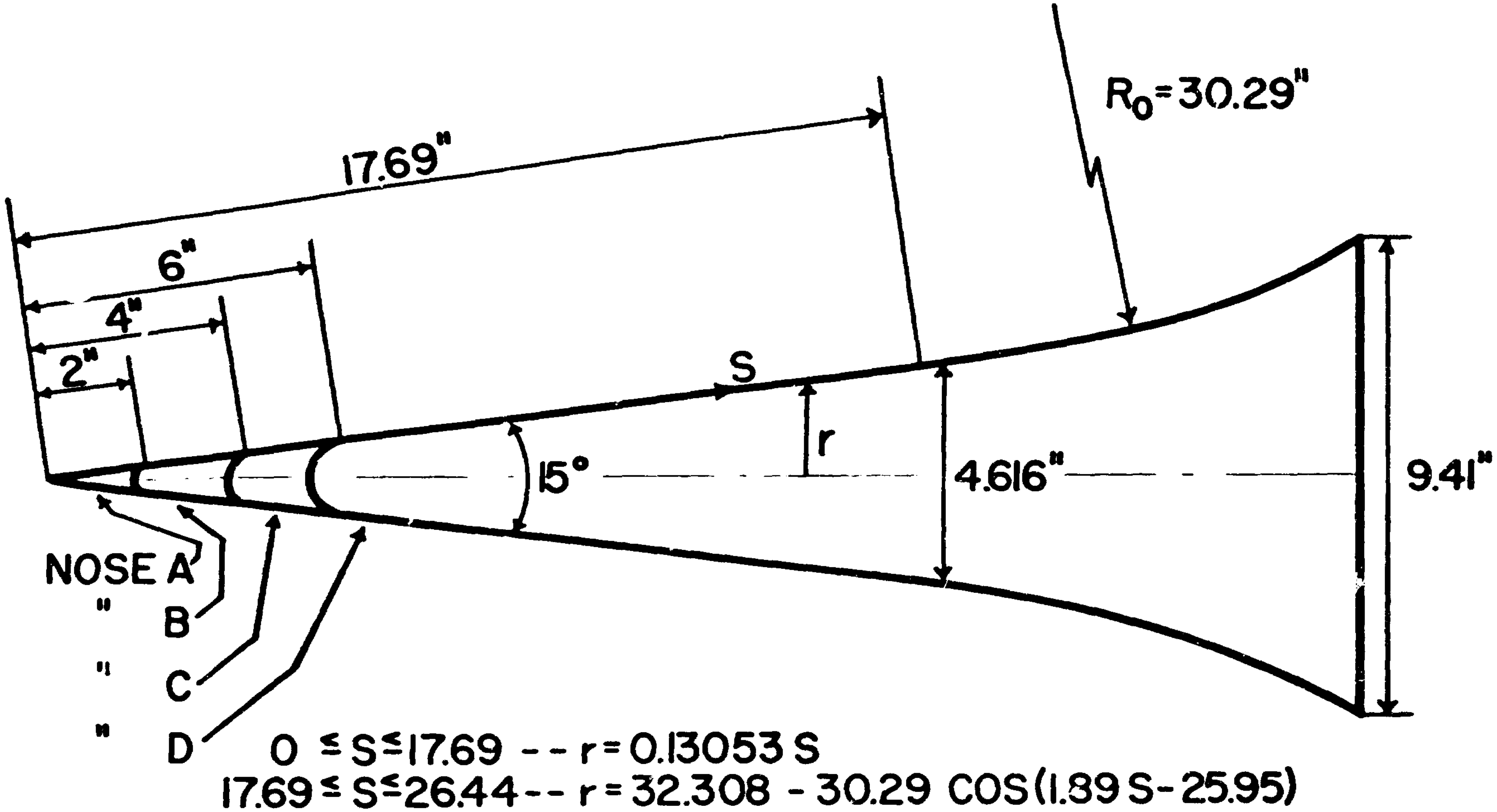
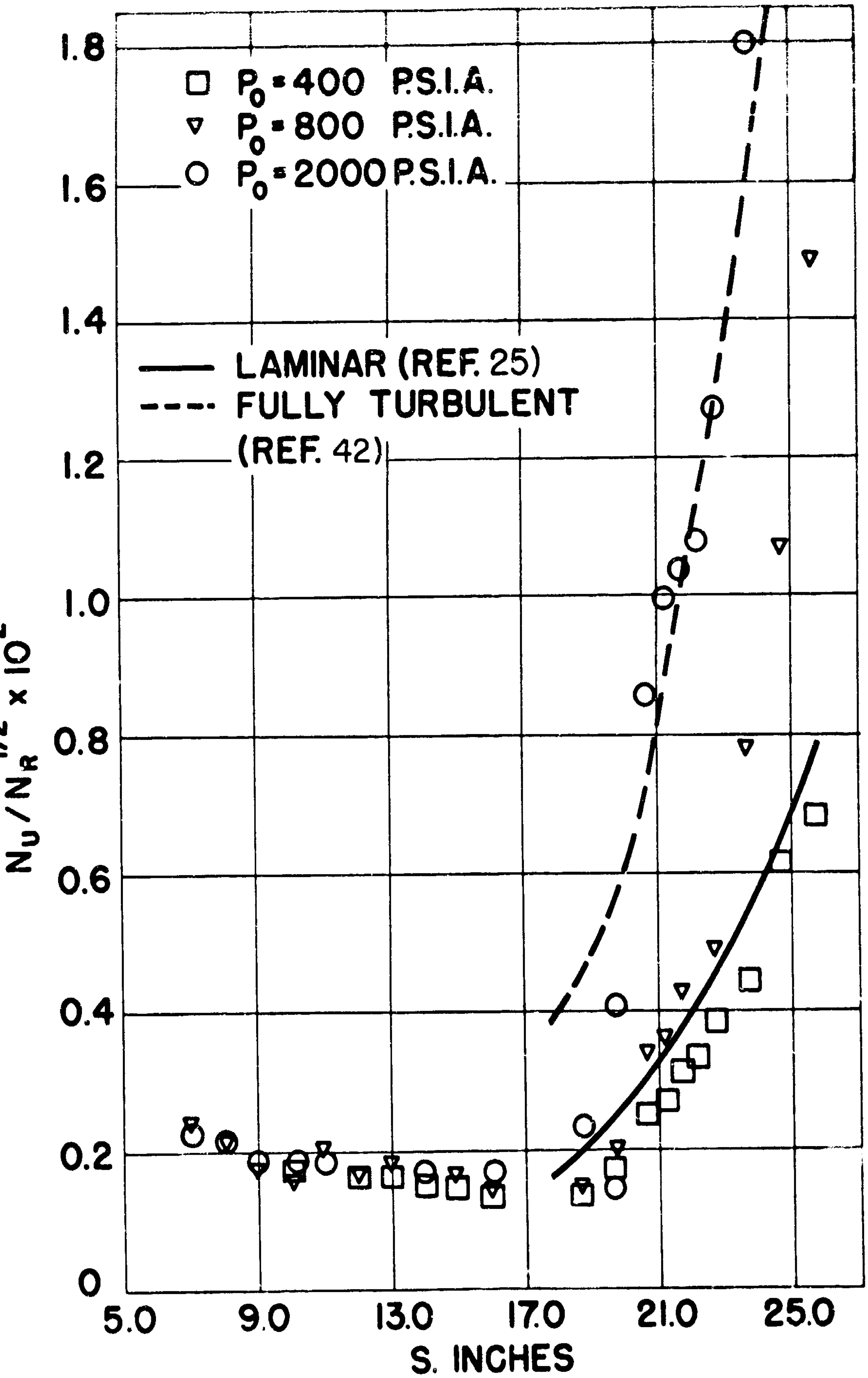


Fig. 11 Law of the Wall for Injection or Suction



NOSE	RADIUS
A	0.032"
B	0.26"
C	0.53"
D	0.80"

Fig. 12 Blunted Cone Flares



Fi [illegible] Heat Transfer Distribution (P = [illegible]

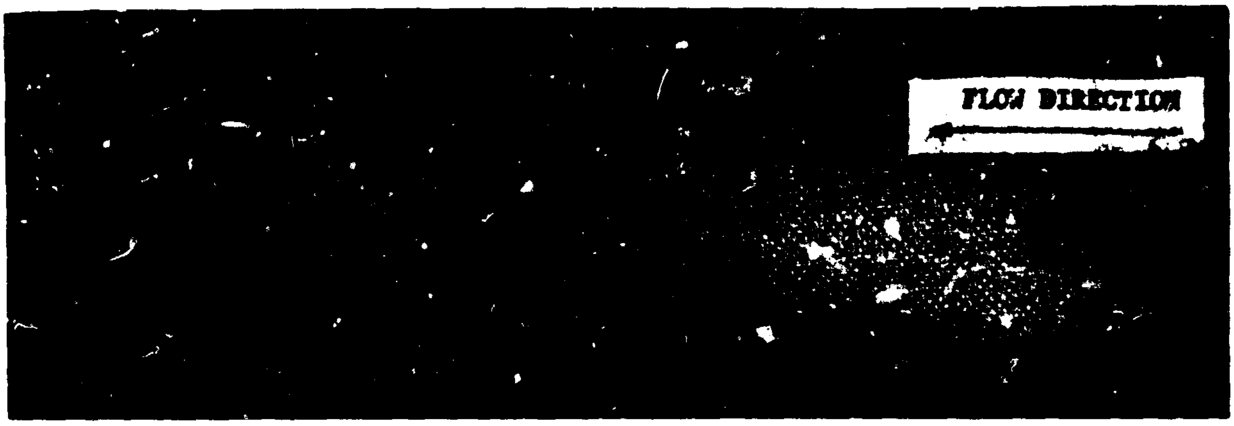


Fig. 14a Photograph of Flow Field, P_o = 400 psia, α = 0°, (Flow is attached)

Fig. 14b Photograph of Flow Field, P_o = 400 psia, α = 0°, (Flow is separated)

Fig. 14c Photograph of Flow Field, P_o = 400 psia, α = 0°, (Flow is reattached)

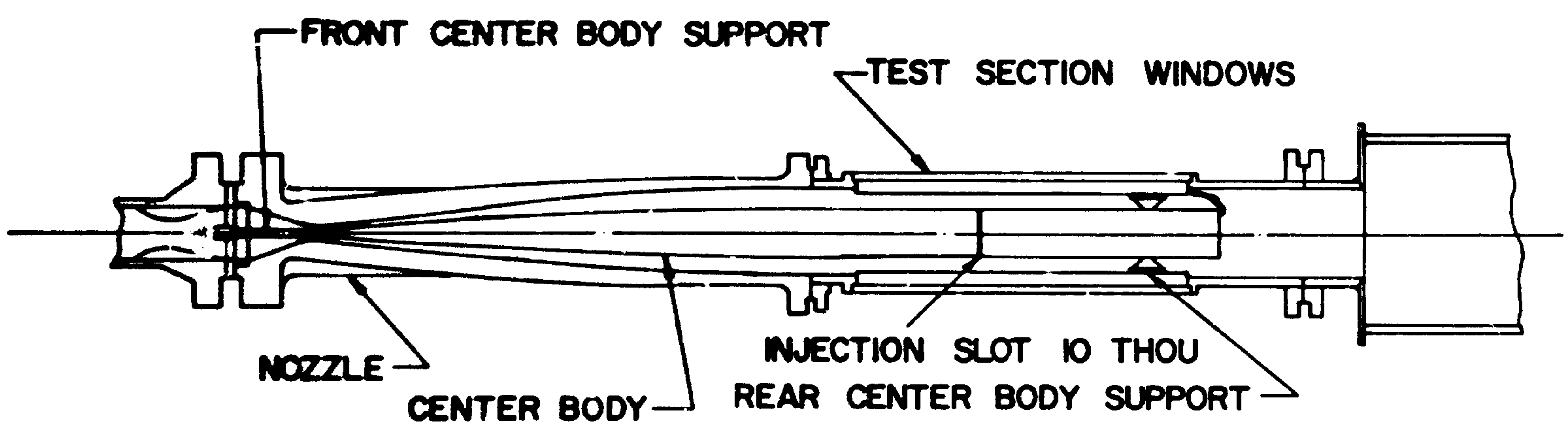


Fig. 15a Streamline Body Installed in Mach 6 Nozzle

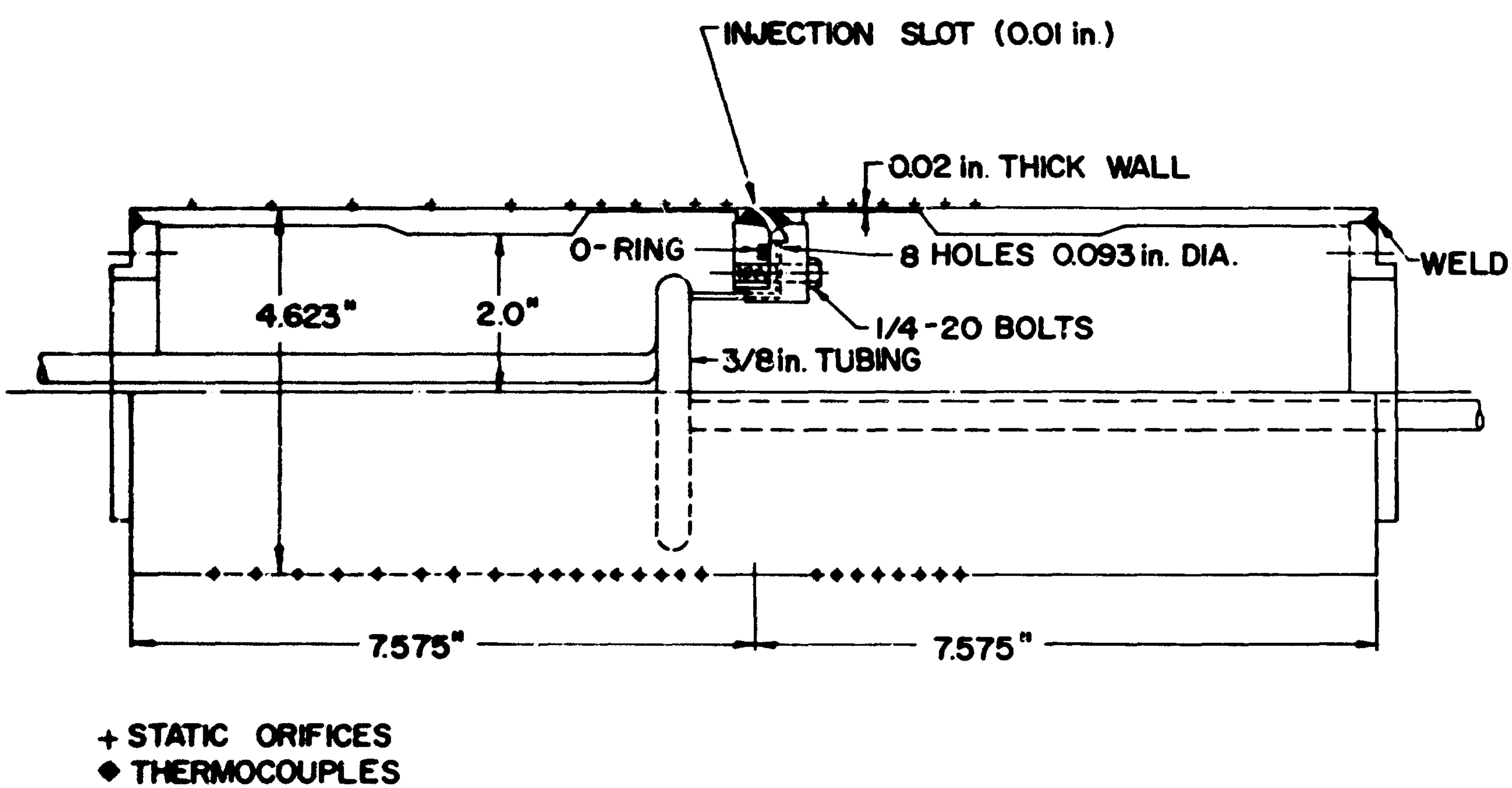


Fig. 15b One Dimensional Model

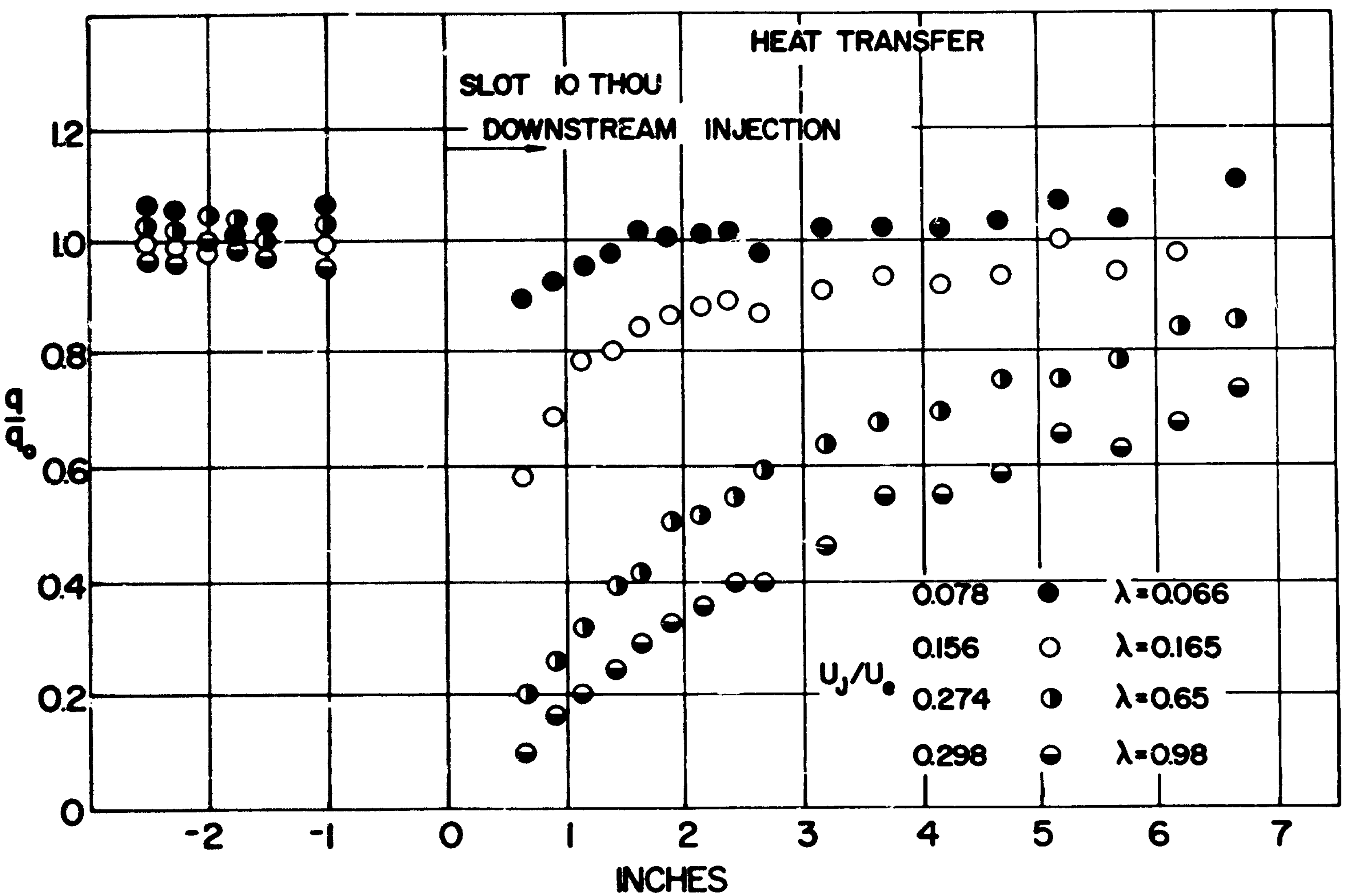
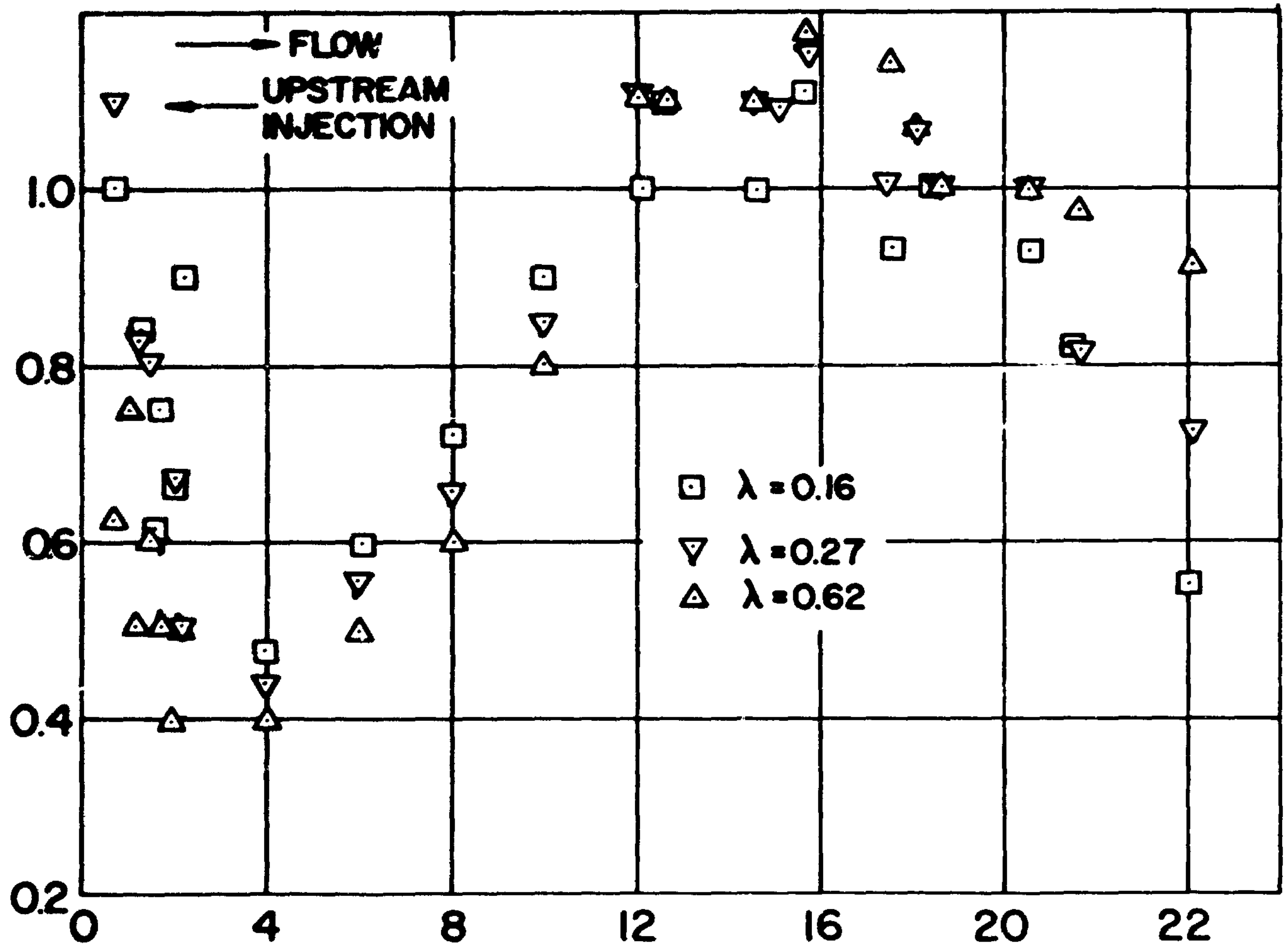


Fig. 16a Upstream Injection Heat Transfer Distribution



Fi.. 16b Downstream Injection Heat Transfer Distribution

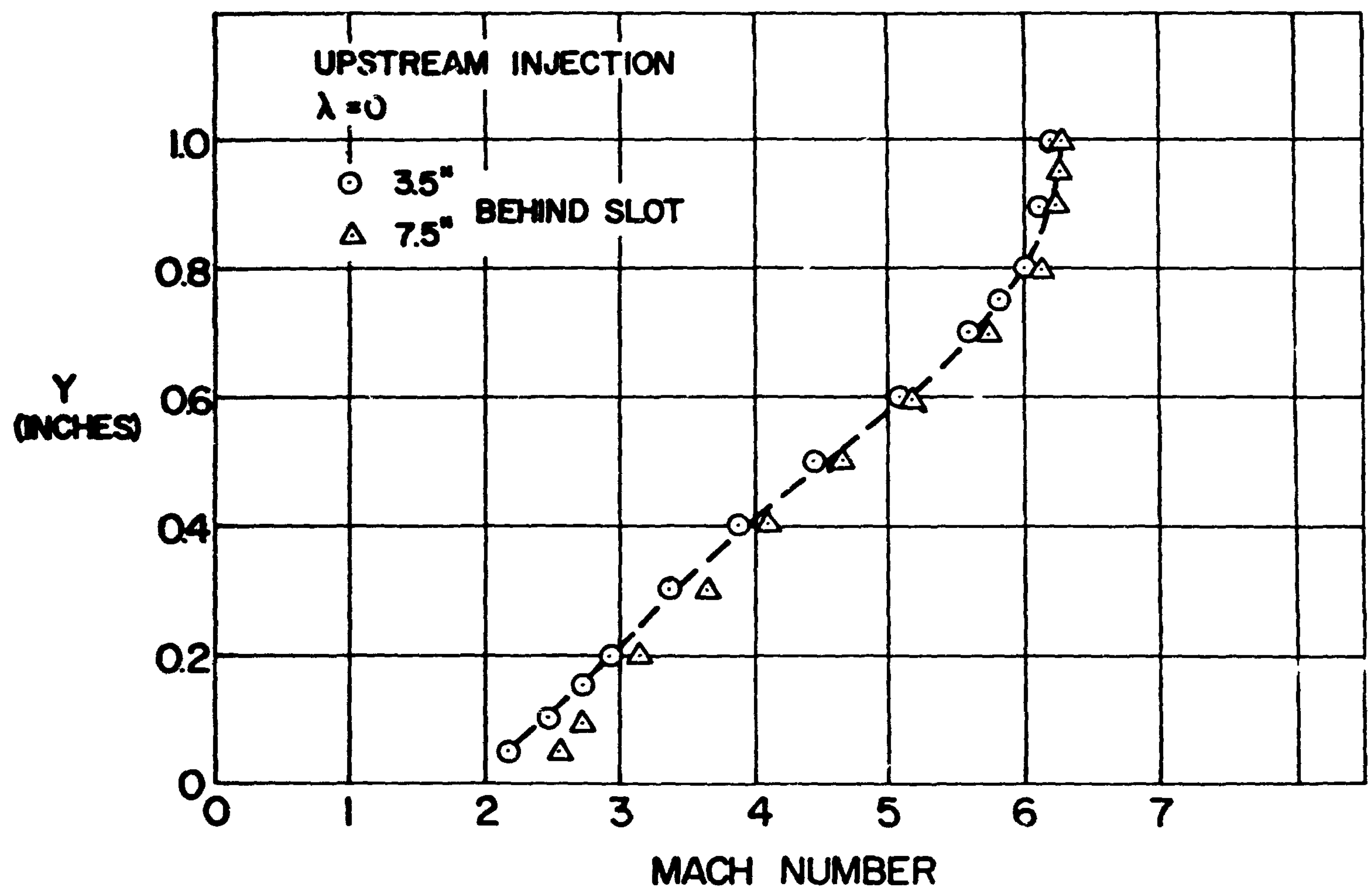


Fig. 17a Mach Number Distribution - No Injection ($\lambda = 0$)

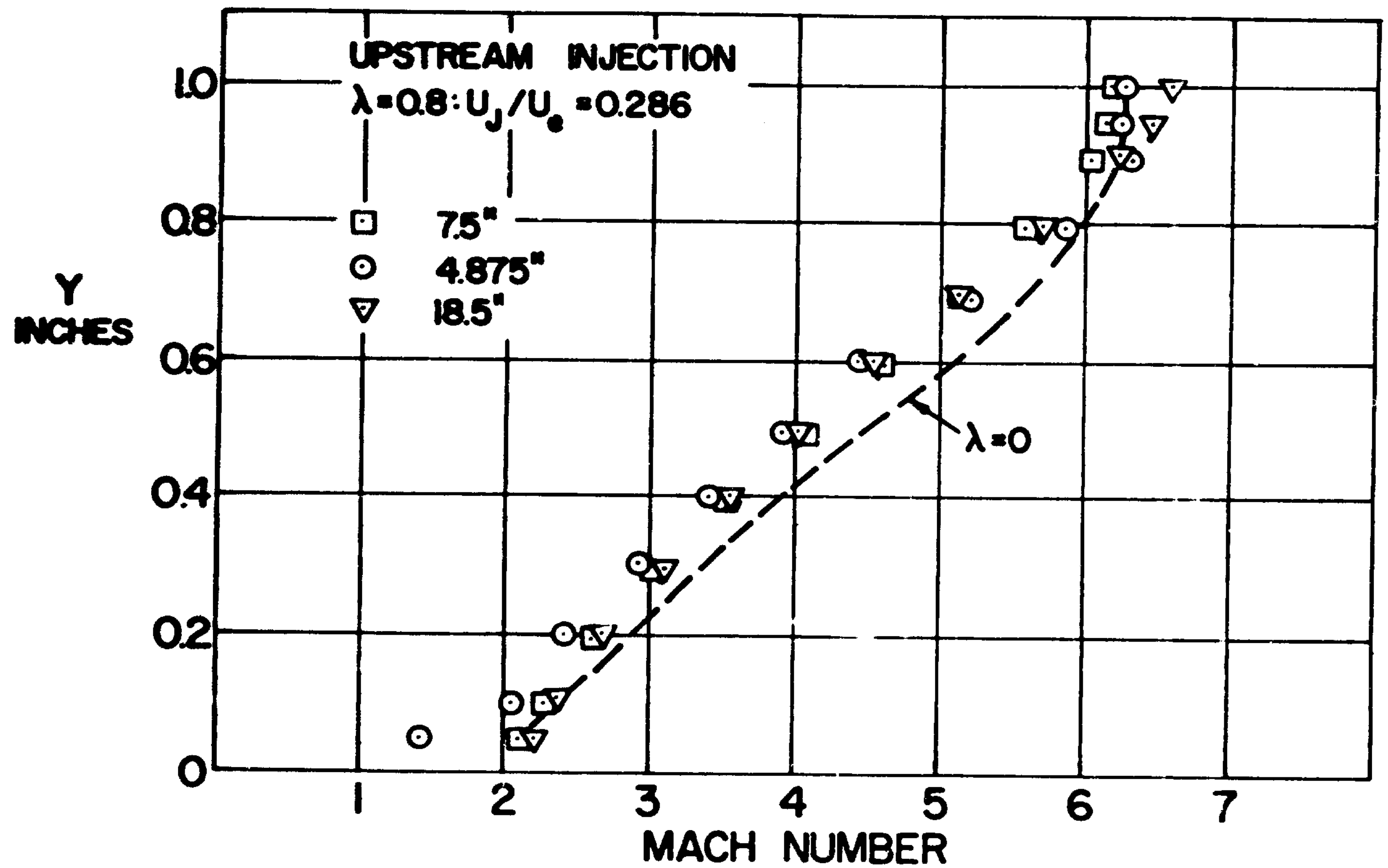


Fig. 17b Mach Number Distribution - Injection (λ = 0.8) and Co.nparison

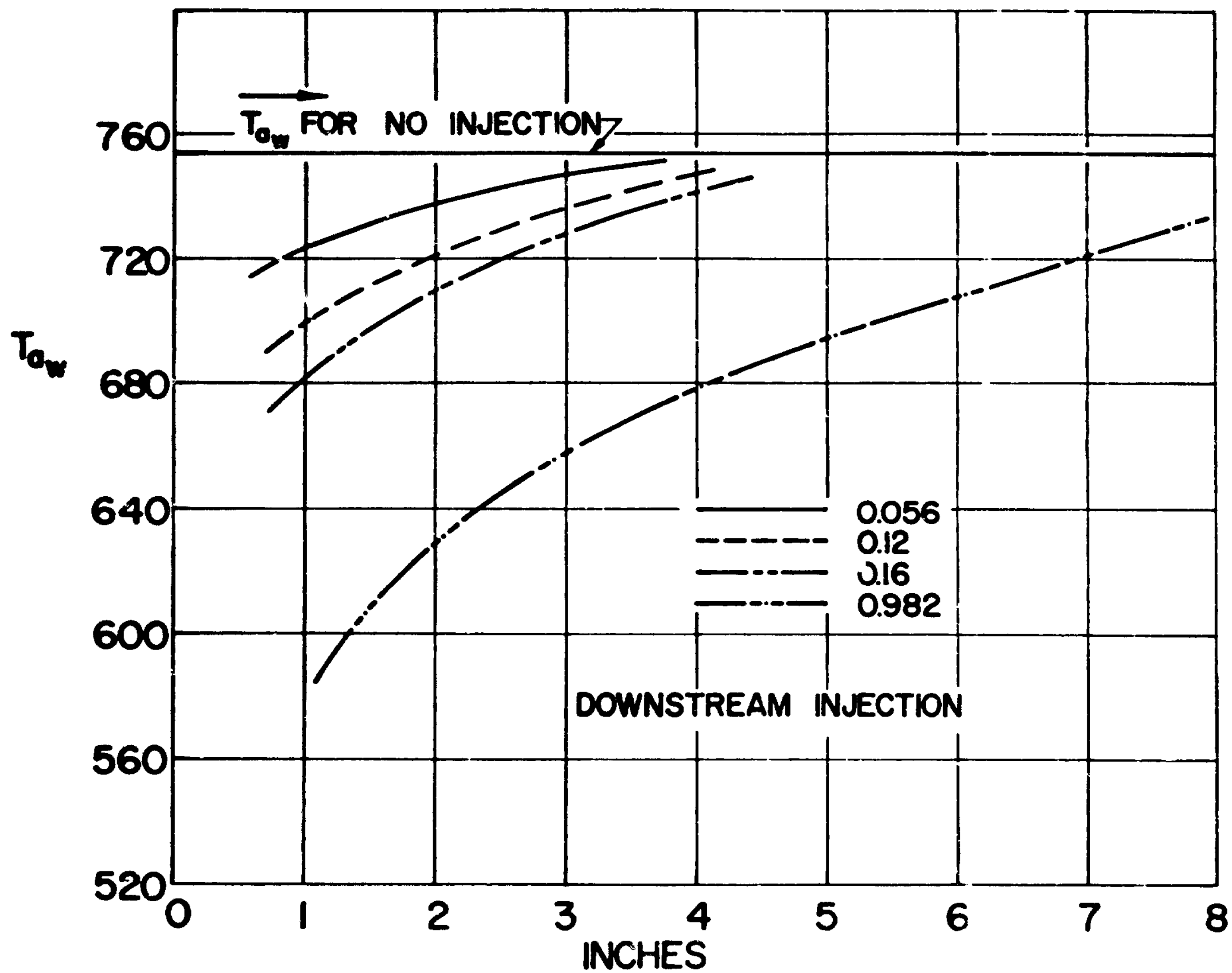


Fig. 18 Correlation for Adiabatic Wall Temperatures with Injection Rates

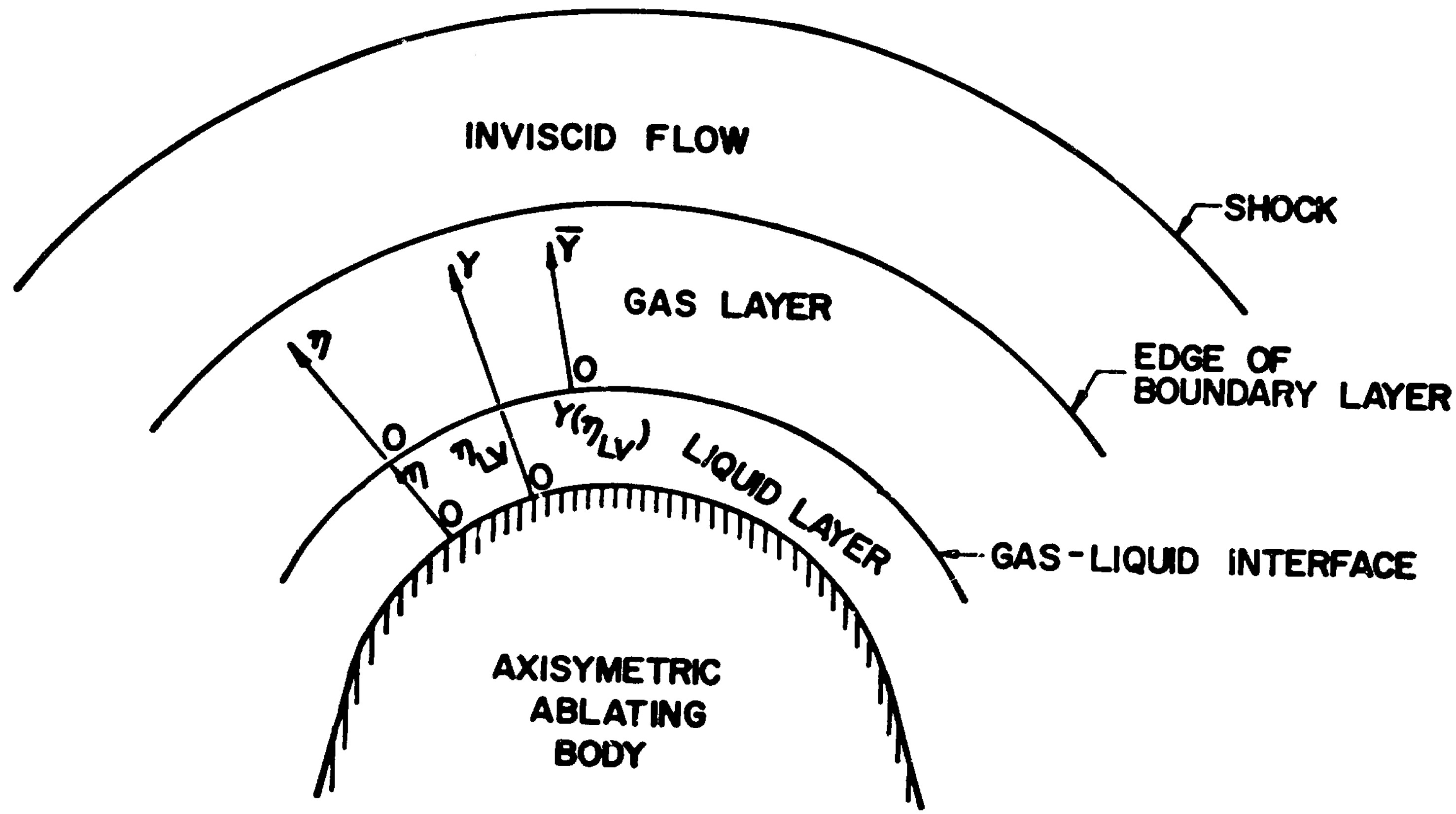


Fig. 19 Schematic Drawing of Flow Field

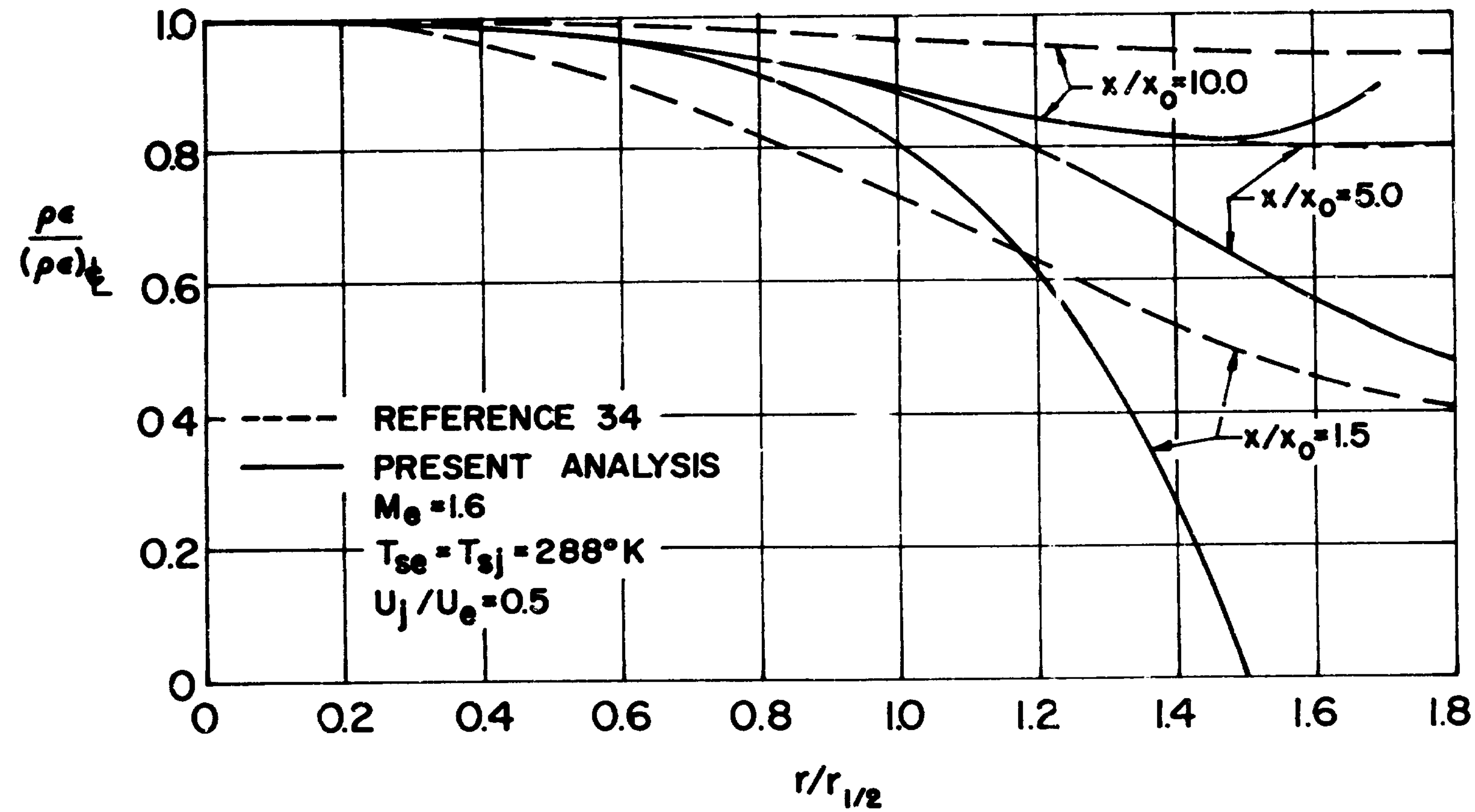


Fig. 20 Radial Variation of the Non-Dimensionalized Eddy Viscosity Coefficient $\rho\epsilon/(\rho\epsilon)_{℄}$ in a Hydrogen Air Mixture

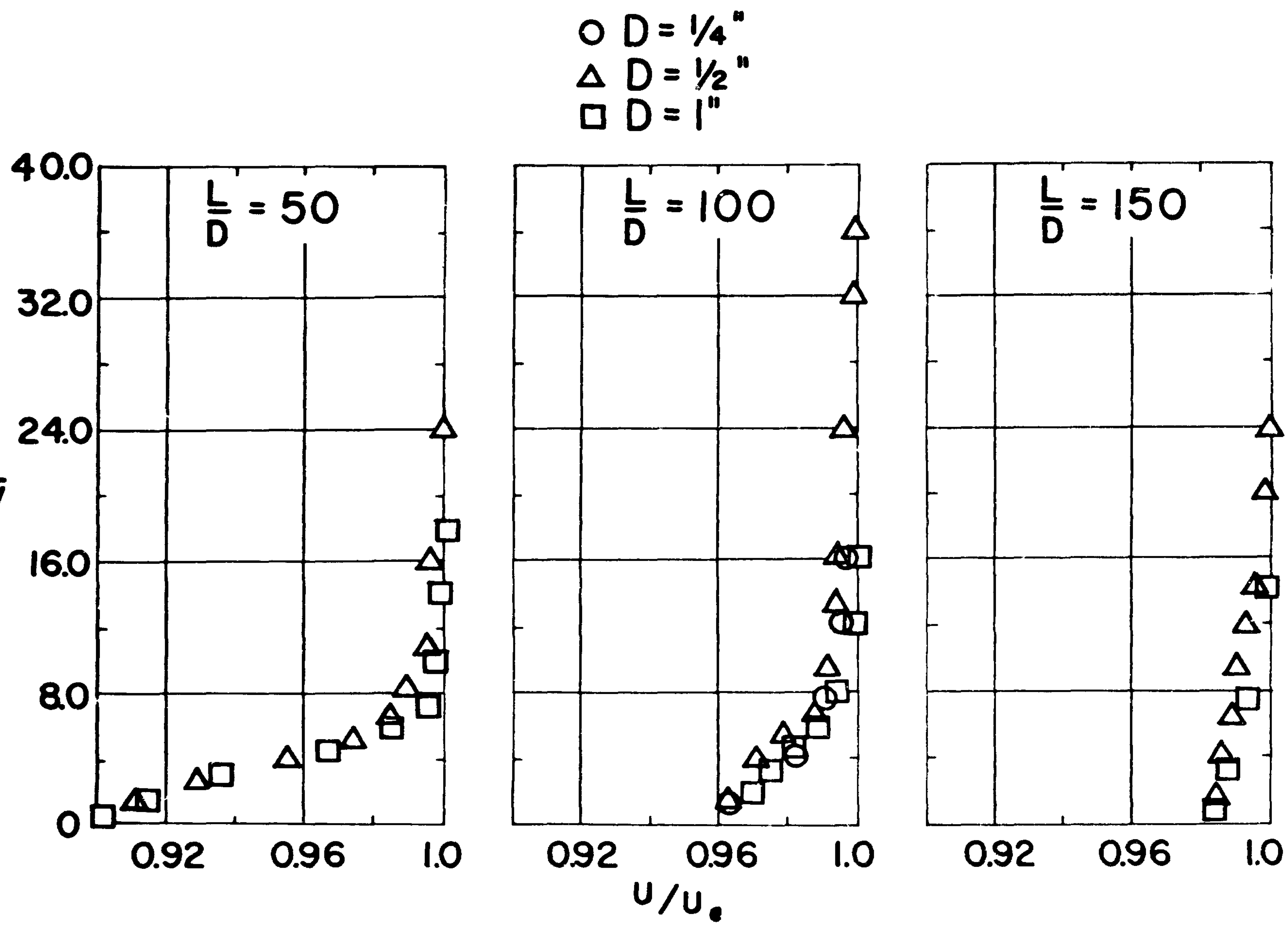


Fig. 21 Velocity Radial Profiles for L/D = 50, 100, 150

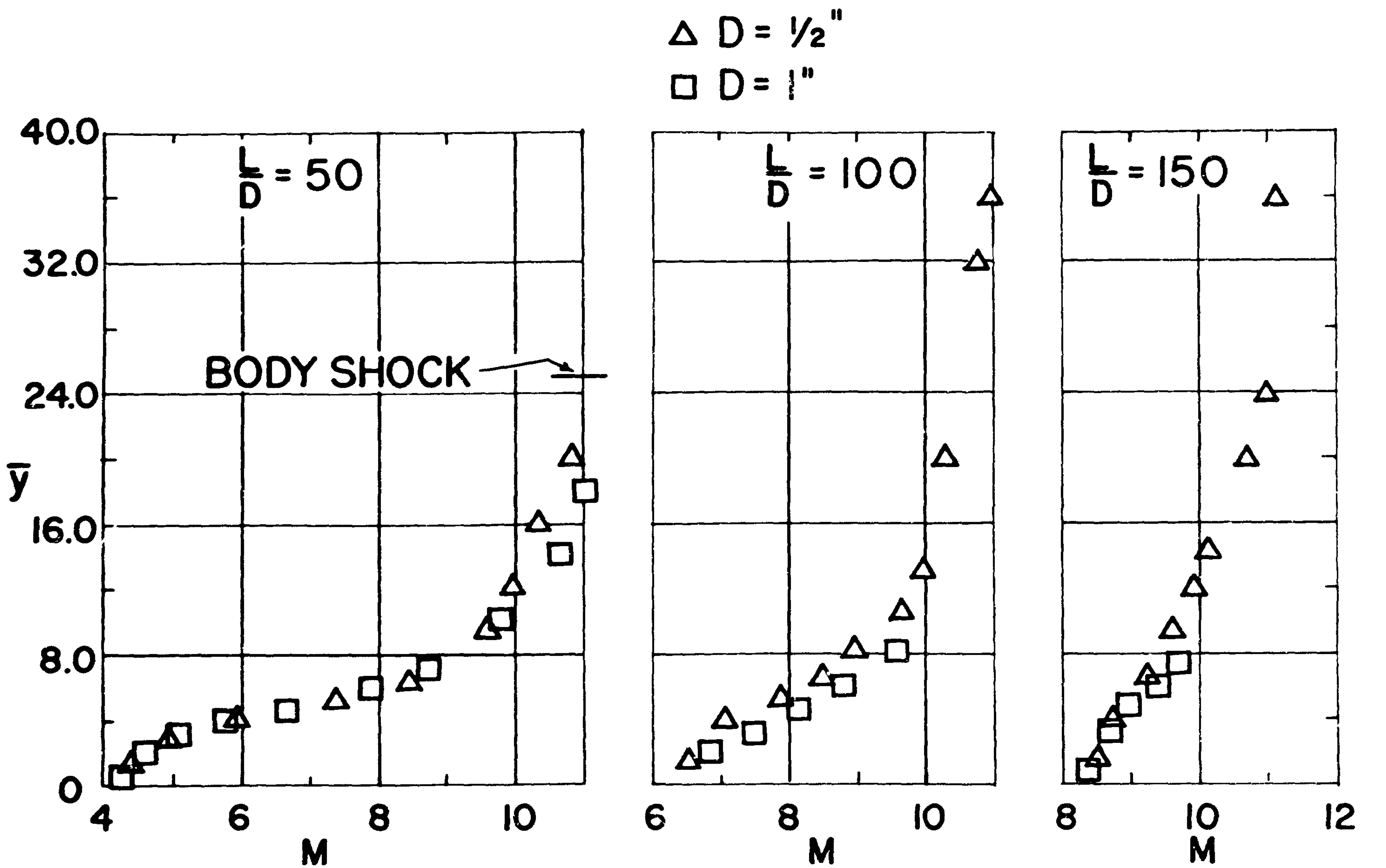


Fig. 22 Mach Number Radial Profiles for L/D = 50, 100, 150

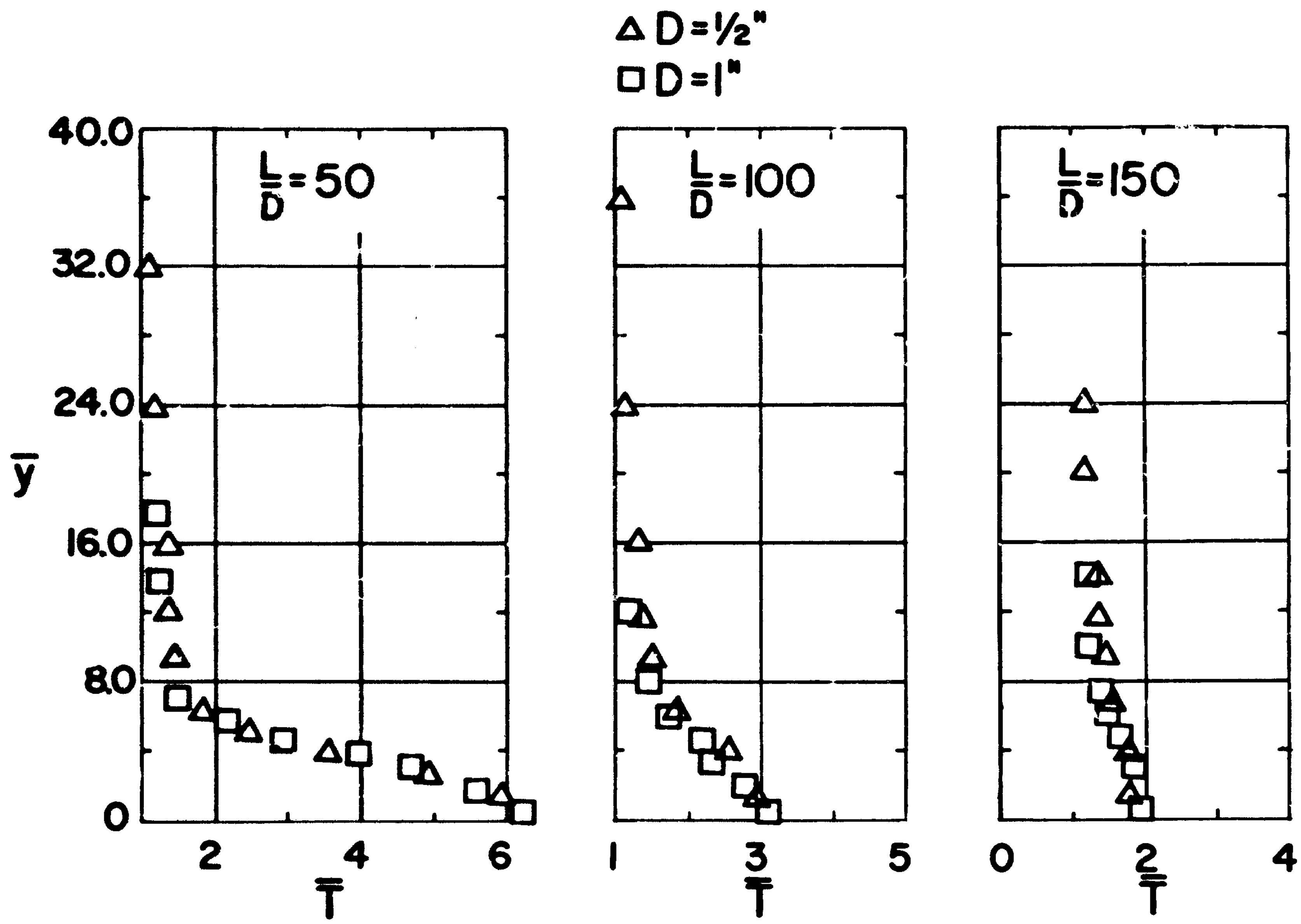


Fig. 23 Static Temperature Radial Profiles for L/D = 50, 100, 150

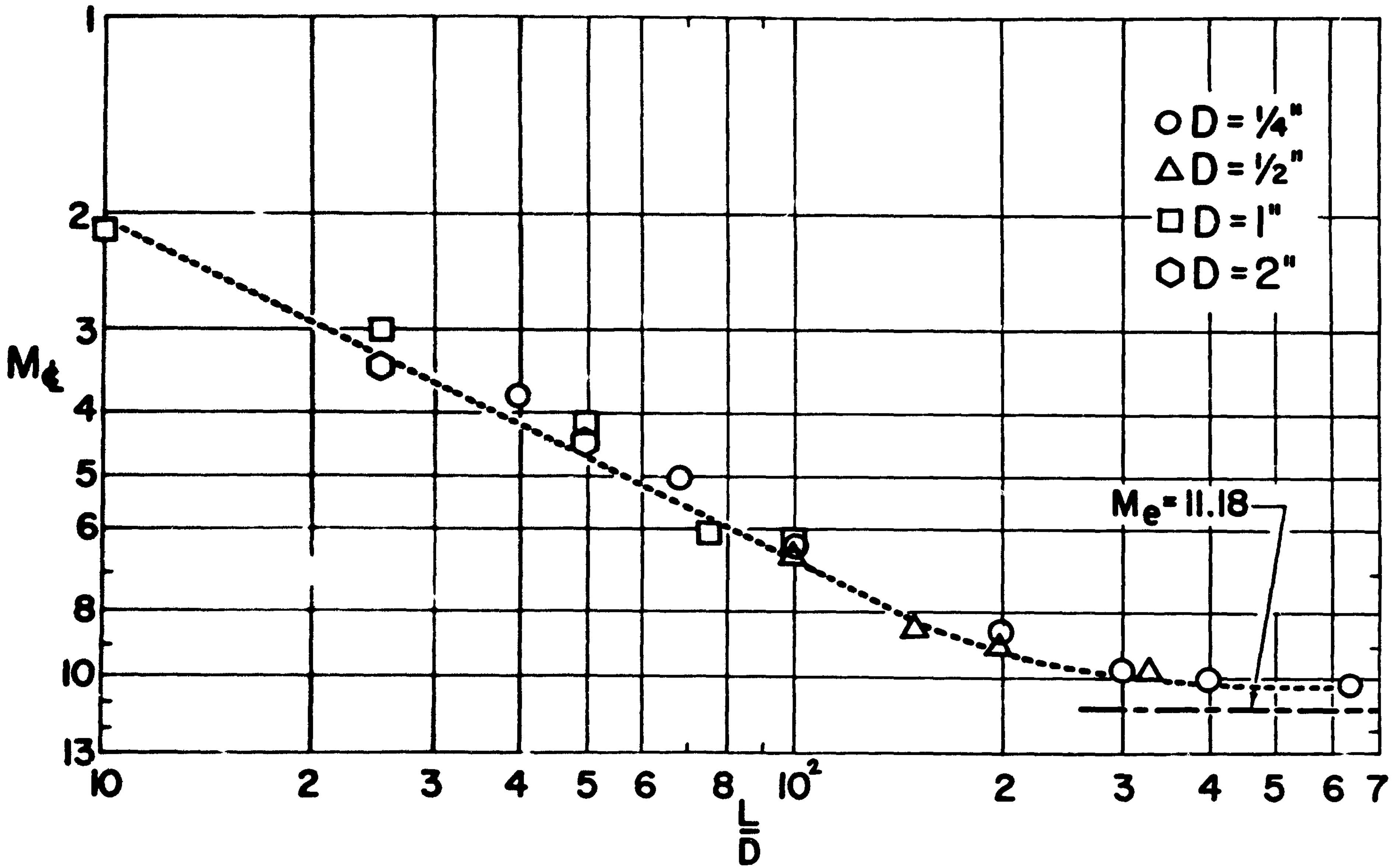


Fig. 24 Experimental Wake Centerline Mach Number Distribution

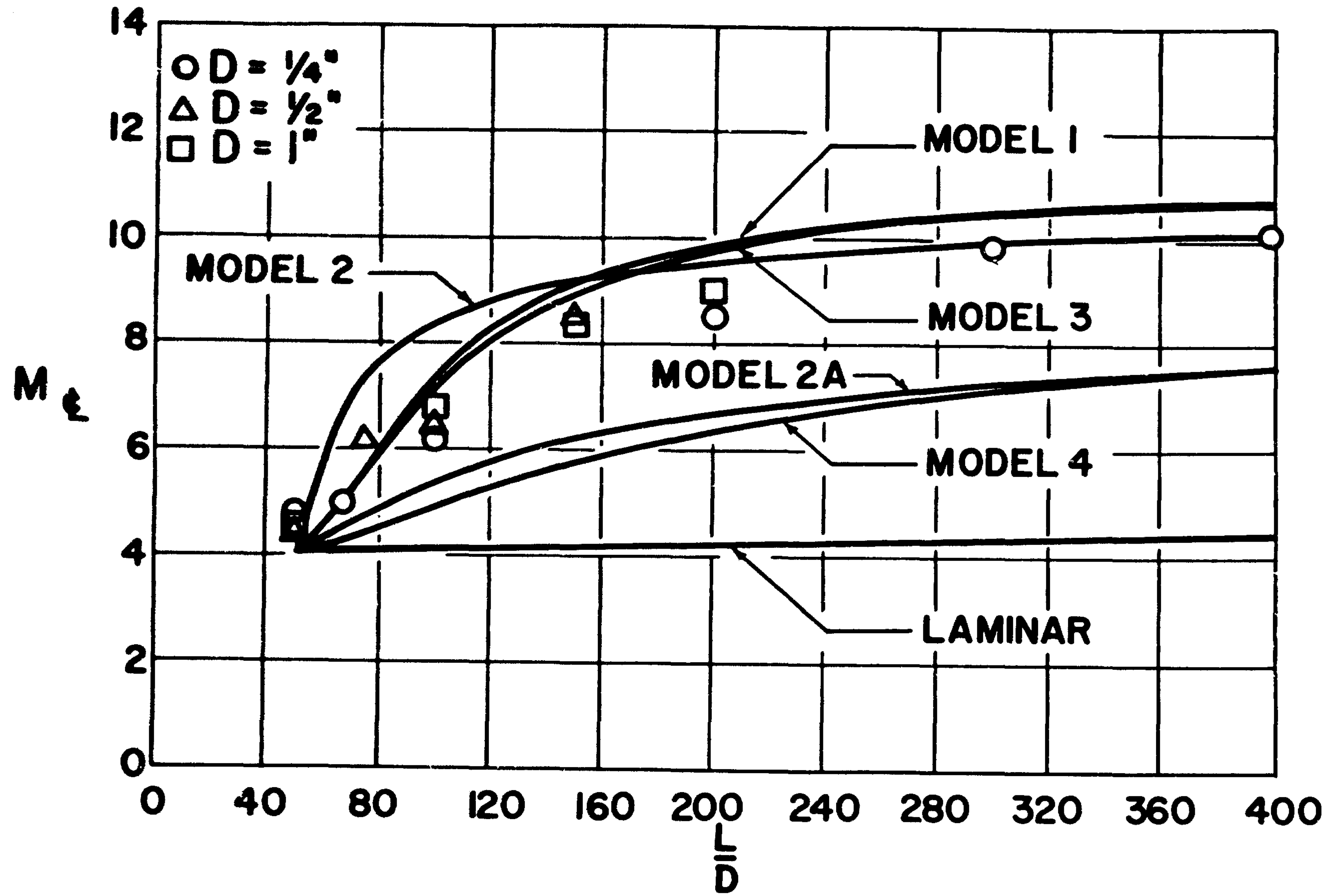


Fig. 25 Theoretical Comparison of Wake Centerline Distribution with Experimental Results: $(L/D)_i = 50$

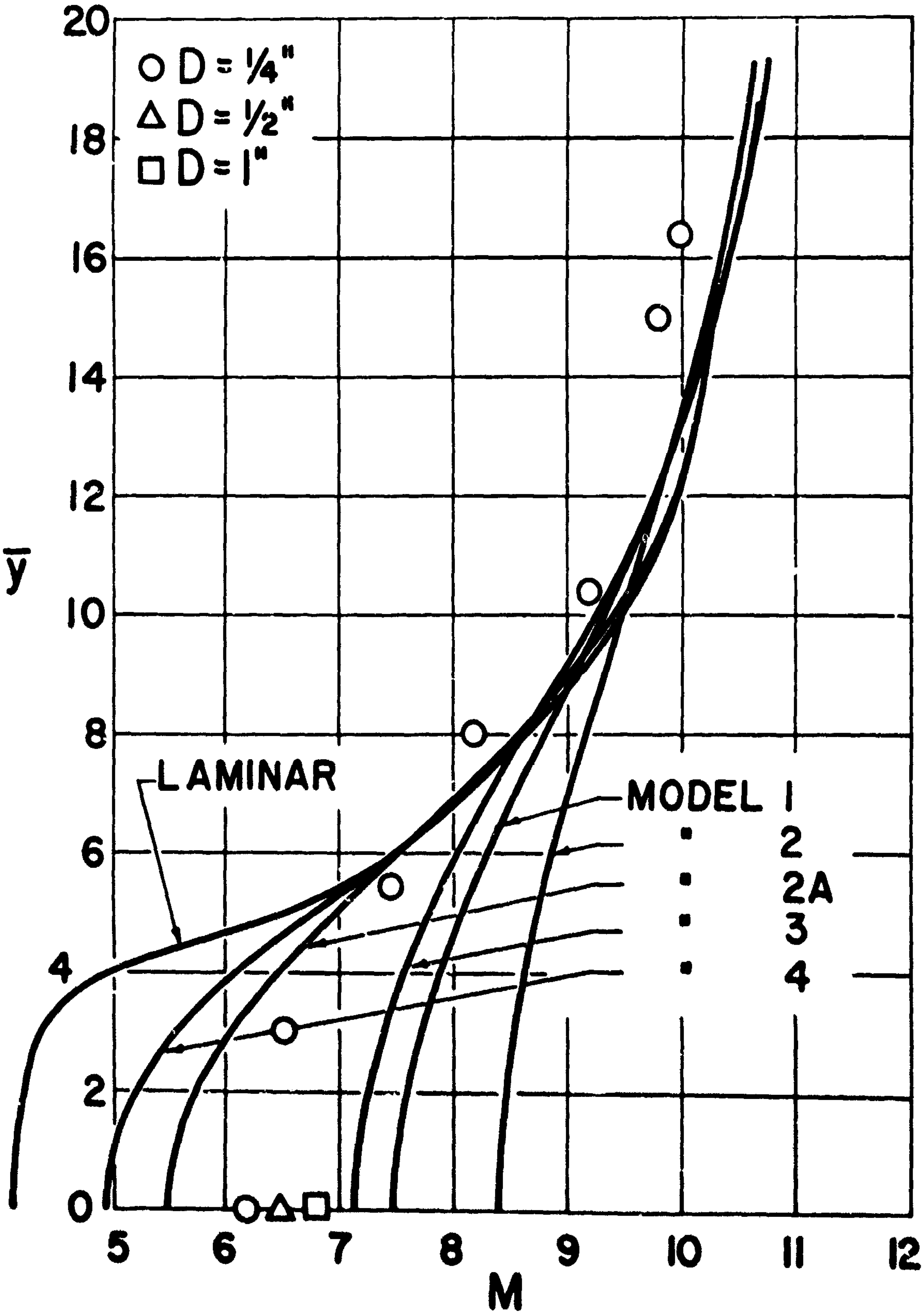


Fig. 26 Theoretical Comparison of Radial Profiles with Experimental Results at L/D = 100

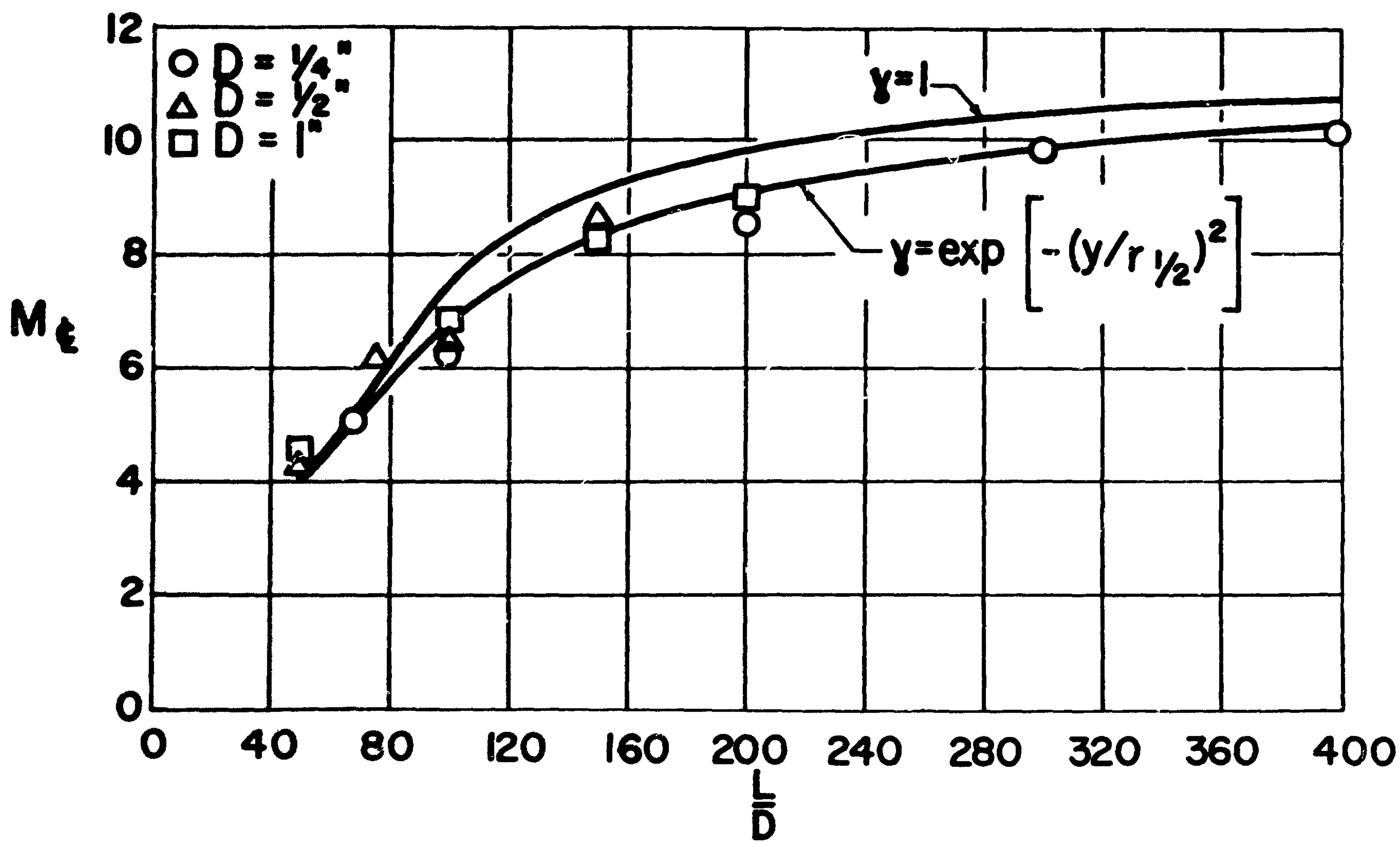


Fig. 27 Effect of Intermittency Factor on Centerline Mach Number Recovery

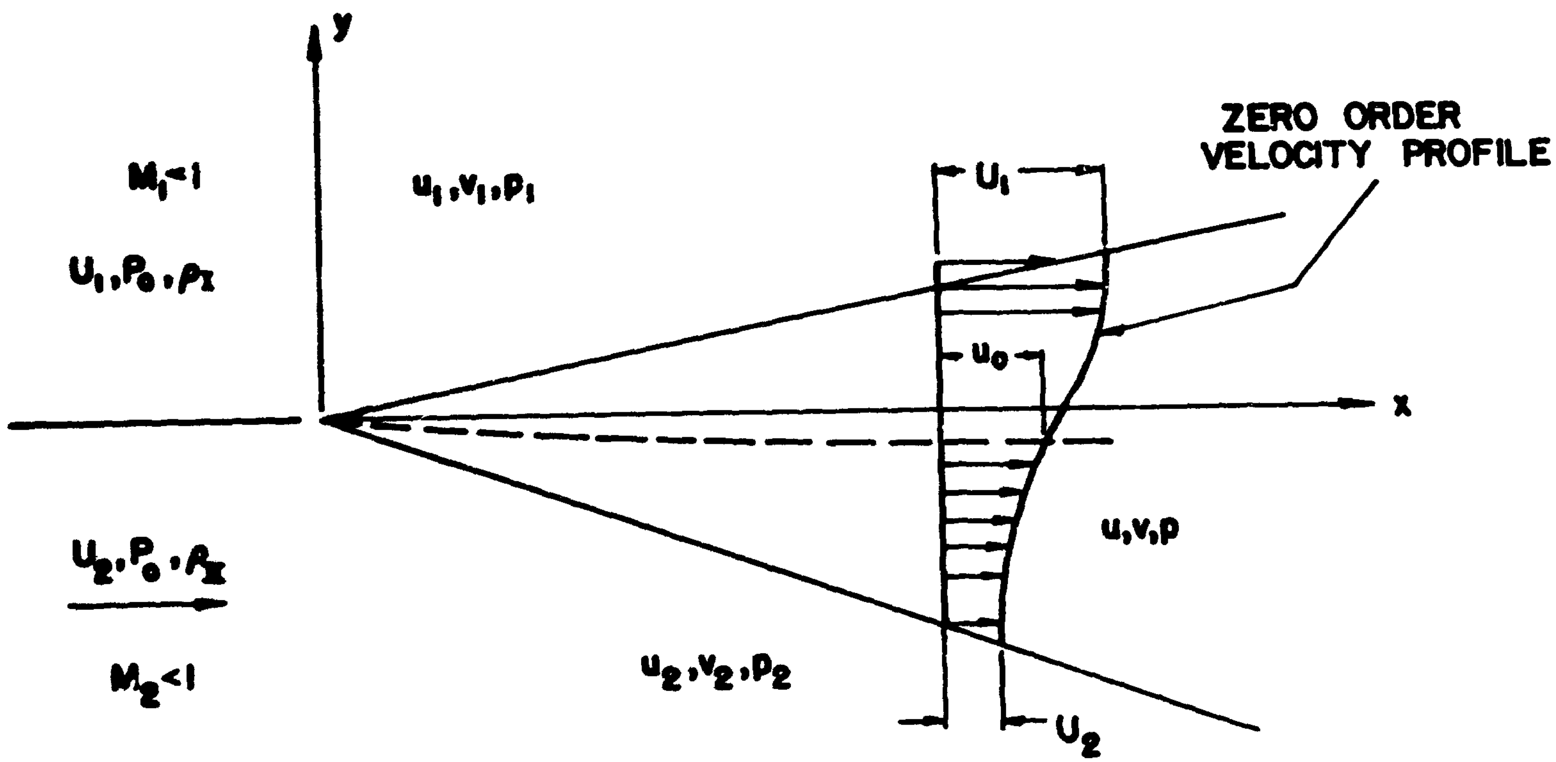
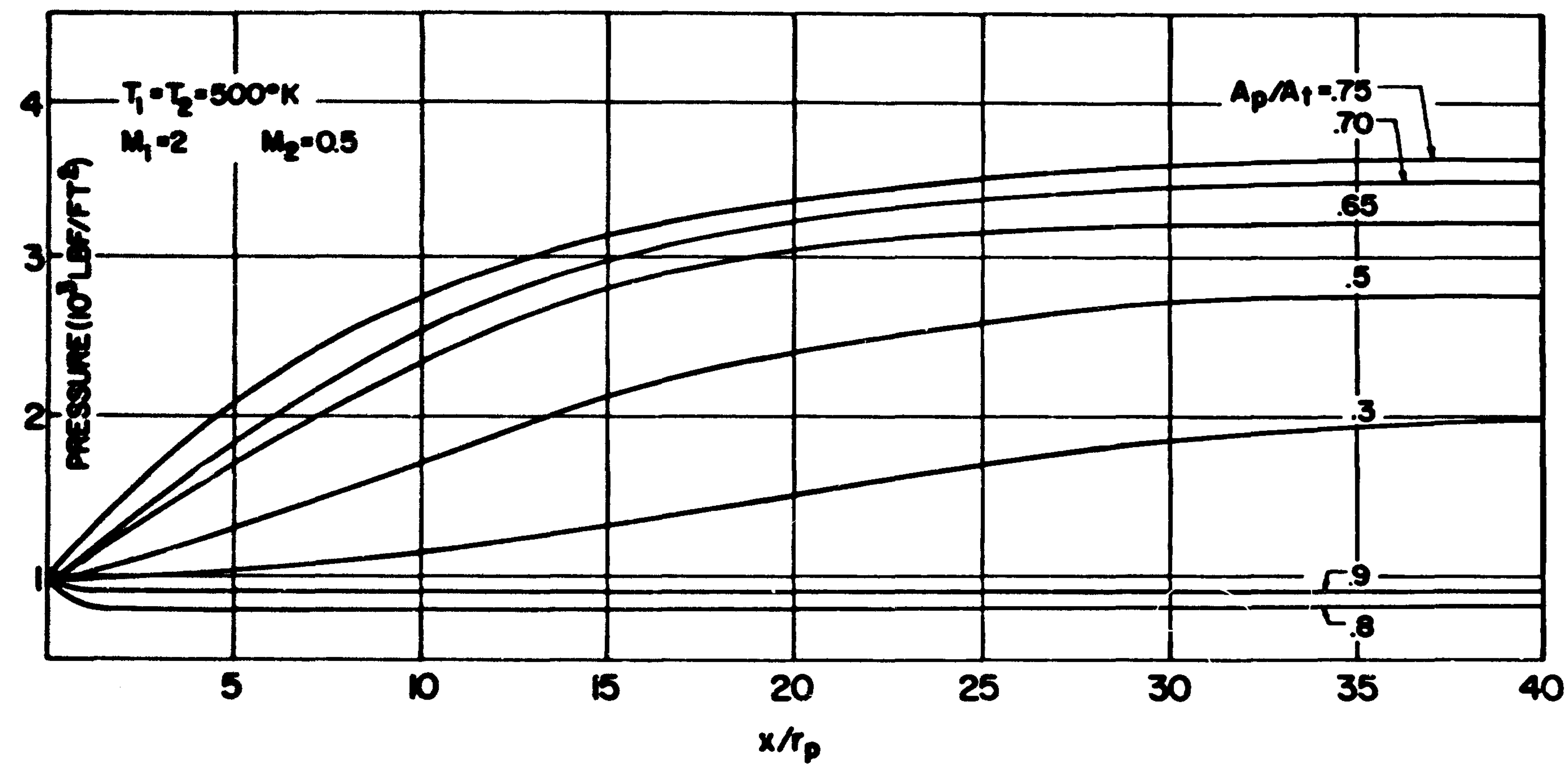


Fig. 28 Mixing of Two Parallel Streams



Fi . 2 Axial Pressure Distribution Air - Air Ducted Flow

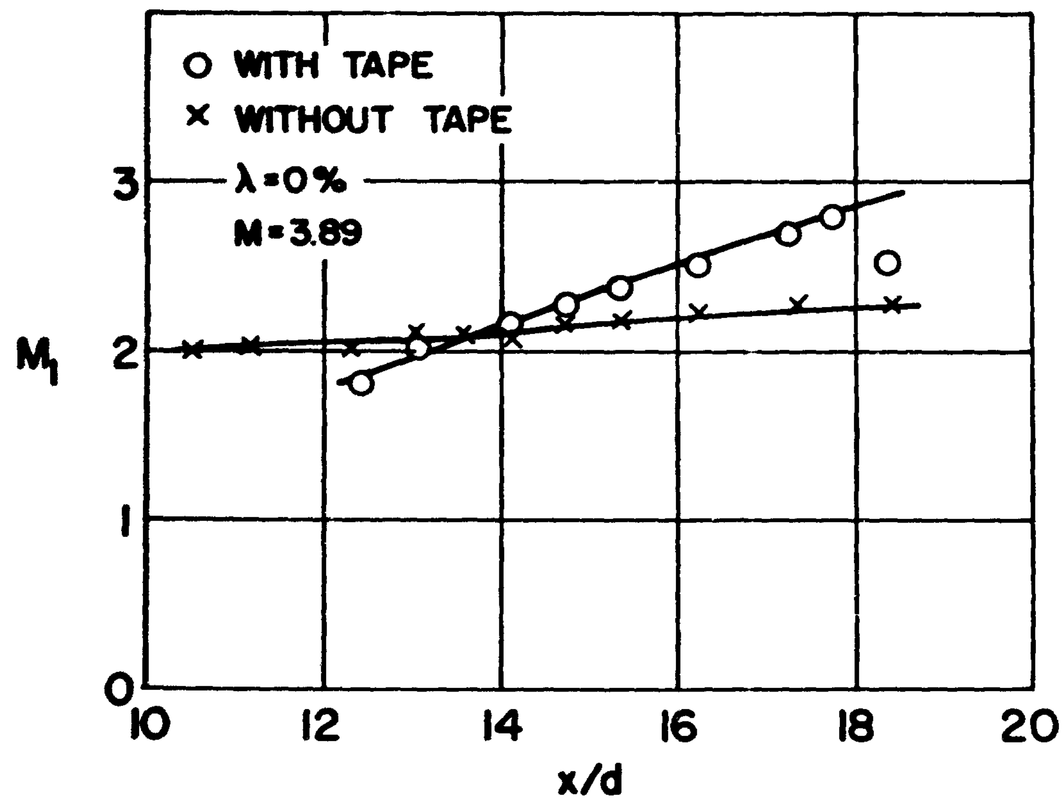


Fig. 30 Mach Number Distribution at the Axis of Wake with or without Small Disturbances

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13. ABSTRACT

Problems connected with viscous hypersonic flow with and without mass addition have been theoretically and experimentally investigated. Specific viscous flow phenomena investigated are: the boundary layer with large density gradients, the mixing in the presence of a wall, and the mixing of two fluids in the presence of pressure gradients. In the field of viscous flow with mass addition, nonsimilar, nonequilibrium, laminar boundary layers with surface ablation of subliming plastic materials have been investigated analytically. In addition, research has been performed in a spectrum of related fields.

The results of the above research have been reported in nineteen technical reports which have been published in the open literature.

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Mass Addition						

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